# Additive Thinking <br> Elementary Mathematics Professional Learning 

## Evidence of Understanding

## +Big Idea \#1

Once students trust "the count", they can flexibly manipulate numbers in order to make solving problems easier by:

- Using Parts and Wholes
- Decomposing / recomposing
- Partitioning
- Compensating
- Using constant difference


## What to look for, what might be evidence of understanding?

- Students manipulate numbers using a variety of strategies to solve questions.
- $7+8=-7+7+1$
- $12+9=10+11$
- $99+24=100+23$
- $324+138=300+100+20+30+4+8$

- Dreambox tool (dreambox.com)

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- Once students work with simplistic numbers, they can transfer that knowledge to larger numbers


## Support

If a student solves a subtraction question only using subtraction strategies, ask them if they could solve it by adding.

If a student does not realize they can manipulate numbers in order to make adding and subtracting easier, you could provide them with ten frames activities that allow them to visualize this process.


## +Big Idea \#2

Students use mathematical reasoning to build connections between inverse problems.
What to look for, what might be evidence of understanding?

- Students manipulate numbers in order to solve similar questions using both additive and subtractive actions.
- Example: Simpler Question: 10-4
- Subtraction: Students may solve it using a variety of strategies, such as with Cuisenaire Rods

Question


NRICH: http://nrich.maths.org/4348

- Addition: Students may solve it using a variety of strategies, such as with Cuisenaire Rods

- Example: More Challenging Question: 104-46
- Subtraction: Students may solve it using a variety of strategies, such as with a number line.

- Addition: Students may solve it using a variety of strategies, such as with a number line.

- Once students work with simplistic numbers, they can transfer that knowledge to larger numbers


## Support:

If a student solves a subtraction question only using subtraction strategies, ask them if they could solve it by adding.

## +Big Idea \#3

Addition is not just adding. It's subtraction as well as it deals with questions where the start, change or result is unknown. It is joining, separating and comparing.

## What to look for, what might be evidence of understanding?

- Students solve a given problem involving a missing addend, minuend or subtrahend, describing the strategy used.
- e.g. Paul has some baseball cards. His friend gave him 4 more. Now Paul has
- e.g. Sylvia had 5 cookies. She gave some to her friend. Now she has 2 left. 42 cards. How many did Paul have to begin with? How many cookies did she give to her friend?


## Support:

Provide students with many opportunities to experience several types of questions. The "categories" provided below contain models showing the variety of question types that students should be exposed to. Students are not expected to identify the category of a question.

- Combine (P - P - W)

Combine (Part - Part - Whole) problems involve distinctly different sets being combined. There is no direct or implied action. You are asked to consider the two SUBSETS as one large set but neither set changes in the process.

- Whole unknown: Sally has $\$ 15$ in bills and $\$ 5$ in coins. How much does she have altogether?
- Part unknown: Sally has $\$ 32 . \$ 15$ are in bills and the rest is in coins. How much is in coin?
- Change (Join)

Change (Join) problems involve two subsets made up of the same items being joined to make a combined set.

- Result unknown: Sally has $\$ 35.25$. She earns $\$ 58.85$. How much does she have at the end of the day?
- Start unknown: Sally has a few dollars. John has \$7. Together they have \$13. How much does Sally have?
- Change unknown: Sally has $\$ 28$. How much more money does she need to save if she wants to buy a $\$ 37$ game?
- Change (Seperate)

Change (Separate) problems involve a large homogeneous set being separated into two subsets made up of the same items.

- Result unknown: Sally has $\$ 57$. She gives $\$ 32$ to pay her mother back. How much money does she have left?
- Start unknown: Sally has some money in her wallet. She spends $\$ 15$ at the store. She has $\$ 41$ left. How much money did she start with?
- Change unknown: Sally has $\$ 28$. She buys a gift. She is left with $\$ 20$. How much was the gift?
- Compare

Compare problems involves a comparison between two quantities.

- Compare Quantity unknown (Type 1): Sally has $\$ 75$. She has $\$ 30$ more than John. How much money does John have?
- Compare Quantity unknown (Type 2): Sally has $\$ 42$. John has $\$ 15$ more than Sally. How much money does John have?
- Difference unknown: Sally has $\$ 5.25$. John has $\$ 3.90$. How much more does Sally have than John?


## Support

Provide students with a variety of tools such as open number lines, Cuisenaire rods, etc. to help them visualize when solving these questions.

## +Big Idea \#4

Subtraction is not just "take away". It is also a comparison (how many more, how many less, what is the difference?).

What to look for, what might be evidence of understanding?

- Students think of the subtraction symbol as "how far apart are they?" or "the space between numbers" in addition to "take away".
- Students can look at a direct comparison to find the difference.
- E.g. Lisa and Mary are growing bean plants in class. Lisa's plant is 7 cm tall. Mary's plant is 12 cm tall. Who's plant is taller and by how much? How do you know?


## Teacher Support

This website further discusses the complexity of finding sums and differences: Top Drawer Teachers

