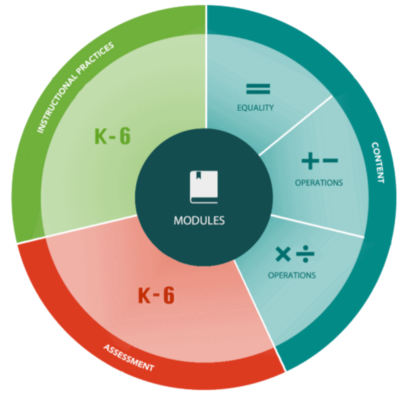
AlbertaRegionalConsortia

Elementary Mathematics Professional Learning

Apprentissage professionnel en mathématiques à l'élémentaire

Understanding the equal sign matters at EVERY grade

As a teacher of mathematics from Kindergarten to Grade 6, have you logged on to the Elementary Mathematics Professional Learnings Opportunities website which can be accessed through http://learning.arpdc.ab.ca/? This site provides you with a variety of resources, materials, professional learning and coaching ideas arranged around 3 big topics:



- *Curricular Content (subdivided into Equality, Additive Thinking and Multiplicative Thinking),*
- *Instructional Practices, and*
- *Assessment.*

These free materials are being developed to explicitly meet the expectations of the Alberta Program of Studies. This article highlights a small snippet of materials drawn from the ***Content Module: Equality****. The examples provided focus on key foundational pieces that students are exposed to in the early grades. We encourage you to visit the EMPLO website to develop a deeper understanding of this topic.*

What is Equality?

In Mathematics, "equal" refers to a relationship not an action or an operation. Equal is used to describe a comparison. We compare amounts, values, and measurement like length, volume, capacity, etc. If a unit is not identified, the comparison between "how much" or "how many" in each set or how much each set is worth. For example, 1 = 10 would be not equal whereas 1 cm = 10 mm is equal because it has units.

What does the equal sign mean?

The equal sign indicates that the items being compared represent the same amount, quantity, value or measure. The equal sign does **not** mean "here comes the answer". The equal sign should be read as "is the same amount as" or "represents the same amount or value as". Once students understand the meaning of the equal sign and expressions, they understand that you can read either side of the equation first. The values are not affected. For example, 3 + 4 = 7 can be read as 7 = 3 + 4. 3x – 2 = 9 can be read as 9 = 3x – 2. Once the meaning of the equal sign has been introduced, teachers in subsequent grades must help students extend and transfer that meaning to new applications. For example 7 x 8 = 8 x 7; 48 = 6 x 8; $\frac{24}{8} = \frac{12}{4}$

Researchers agree that by introducing students to the relational meaning of the equal sign as in 9 = 9 or 5 = 5 before they are expected to write equations, teachers can reduce the potential for students developing misconceptions. (Capraro et al., Knuth et al., Powell, Watson et al.,) Teachers and textbooks often only represent equations in the form of 6 + 2 = 8. The left side has the "operations" while the right side has "the answer". To help students develop a more complete understanding, teachers should consider how often they expose students to equations written as 8 = 6 + 2.   In this case, the left side has the "answer" and the right side has the "operations." A simple idea to think about: Every time students read or write an equation, ask them to state it in another format. For example, a student says 3 + 2 =5. How many ways could you record that equation without changing the meaning? 2 + 3 = 5; 5 = 3 + 2; 5 = 2 + 3. Look at all the practice!

Why is Equality Important?

The understanding of equality as a relationship forms the basis for all number properties. The understanding and application of number properties is a significant factor in students building efficient strategies for computation and forms a foundation for success in algebra.

However, if you asked your students to explain what the equal sign means, what would they say? Current research confirms that students who demonstrate a correct understanding of the equal sign show the greatest achievement in mathematics. (Capraro et al., 2007). When asked to explain the equal sign, the majority of North American students, across the grades K-12, are apt to say "It means here comes the answer." This makes it difficult, if not impossible for learners to accept equations in the form 7 = 7; 8 = 3 + 5; 3 + 4 = 2 + 5; or x = x.

Why begin with Equality?

It is important that students first understand relationships between numbers before they are asked to operate with them. Operations emerge as a thinking and communication tool to help us in our quest to determine and prove equality and inequality. The equal sign naturally arises through a need for the formal presentation of determining equality.

During a research study conducted by Falkner et all (1999), students were asked to make the following statement true: 8 + 4 = __ + 5. Student responses varied and many were simply incorrect.

1. Some created a "run-on" equation. 8 + 4 = <u>12</u> + 5 <u>= 17</u>

This is incorrect thinking and notation because 8 + 4 does not equal 12 + 5. The convention is that there should be one equal sign per equation. One exception would be an example such as 1 + 5 = 2 + 4 = 3 +3. In this case, all three expressions are equivalent.

2. Some wrote 8 + 4 = 12 and then crossed out the + 5, claiming the author made a mistake in the print. Imagine a student in junior high applying this error to a question such as 3x + 4 = 2x + 9!
3. Some changed the symbols to 8 + 4 + 12 + 5 = 29, claiming the author forgot to finish the equation.
4. Some understood equality as a relationship and realized that 8 + 4 = 7 + 5.

Undoing a misconception is way more work than teaching it well the first time.

What's a Teacher to Do?

In Kindergarten and Grade One, set up opportunities for students to compare 2 sets as equal as students work on basic recognition of quantity.

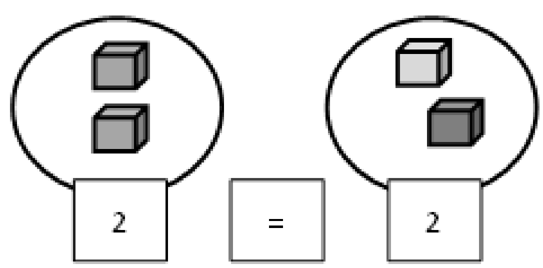


Fig. 1

In Fig. 1, the teacher places the blocks down at the same time in each set. Students identify how many in each set. The teacher replaces the blocks with the numerals and places the equal sign between the numerals. The last step is to read the equation out loud. Two equals two. Two is the same amount as two. The left side equals the same value as the right side. The left set has the same amount as the right set. Practice with this idea should include maintaining the same quantity but varying the items in one of the sets. For example, 2 blocks = 2 bingo chips.

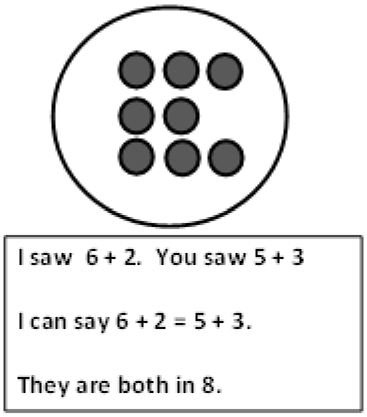
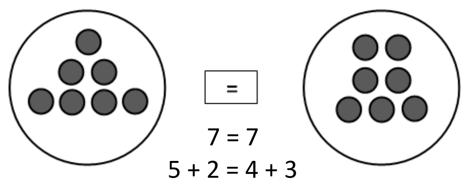


Fig. 2

As students in Grades 1 to 3 develop fluency with numbers to 20, their practice can include a variety of equations that describe equal such as those in Fig. 2. Practice should include explaining and writing equations in a variety of ways. Activities like this help students recognize that an expression can equal an expression.

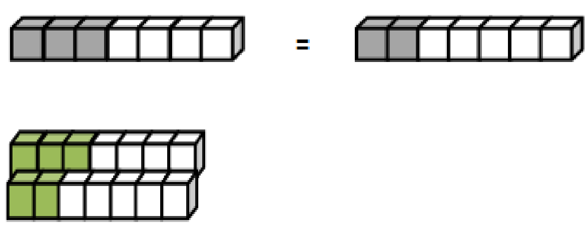
In Fig. 3 students are comparing two equal sets rather than a single set as in Fig. 2.   In this example, students were presented with the model and asked to create the equations.



7 = 7
5 + 2 = 4 + 3
7 = 4 +3
3 + 3 + 1 = 7
Fig. 3

Ask students to help you use materials to “prove” whether or not equations are true. In Fig. 4, students were presented with the equation (an equality) and asked to prove it with a model.

3 + 4 = 5 + 2



How do these students use models to prove the equation?
Fig. 4

The understanding of equality as a relationship forms the basis for all number properties. From Grade 2 to Grade 6, the understanding and application of number properties is a significant factor in students building efficient strategies for computation. For example, building multiplication facts as area models is one way to demonstrate 3 x 4 = 4 x 3. In Fig. 5, students can manipulate their models to explore the idea of the commutative property as an equality.

Fig. 5:
The array for 3 x 4 also represents 4 x 3.
They cover the same area. 3 x 4 = 4 x 3

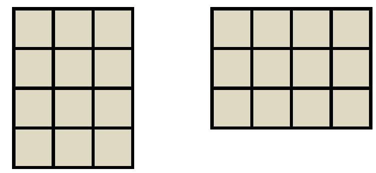


Fig. 5

Another significant number property is the distributive property which is introduced in Grade 4. In Fig. 6, students can manipulate an area model to explore the idea of the distributive property as another demonstration of an equality.

Fig. 6:
The array for 7 x 8 is used to demonstrate 7 x 8 = (5 + 2) x 8

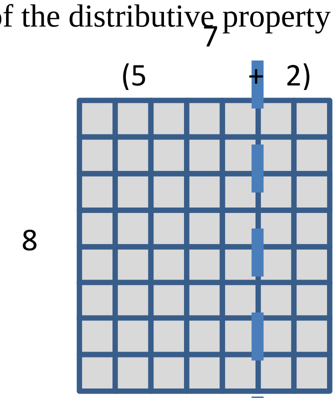


Fig. 6

The Balance Metaphor

At Grade 1, the curriculum states that students be introduced to the metaphor of balance in the Patterns and Relations outcome number 4: “Describe equality as a balance and inequality as an imbalance, concretely and pictorially (0 to 20)”. Teachers need to be cautious as students’ understanding of weight may interfere with their ability to consider whether or not 2 sets or expressions balance. When 2 lengths or 2 volumes are being compared, the metaphor “balance” has no meaning. We really use balance when we’re talking about equations that are balanced. In order to address all aspects of equality, teachers need to use a variety of models and/or metaphors. When using the balance metaphor, we want to ensure students understand that we use the term as an adjective to describe the state of equality rather than a verb.

Note: the Grade 1 Achievement Indicators suggest that students using a balance:

- "construct two equal sets, **using the same objects** (same shape and mass), and demonstrate their equality of number, using a balance (limited to 20 elements)."
- "construct two unequal sets, **using the same objects** (same shape and mass), and demonstrate their inequality of number, using a balance (limited to 20 elements)." (Alberta K-9 Mathematics Achievement Indicators, 2016)

The balance metaphor is one way to develop strategies for setting up and solving equations. Working with balances (as in Fig. 7) allows students to practice number facts as they learn what it means to "balance equations".

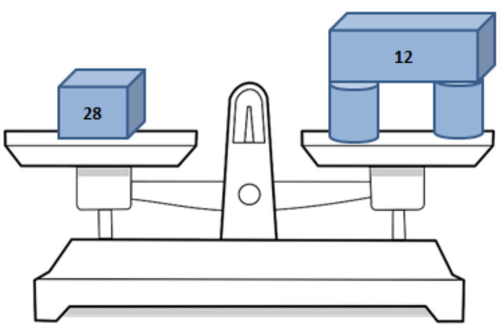
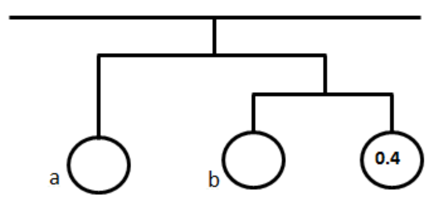


Fig. 7

Where would you start to decide what numbers are missing on the right hand side of the balance in Fig 7?
Student A: I am thinking 28 = 12 + something x 2
Student B: I am thinking subtract the 12 on the right and 12 from 28 and you will still have equal. I think I would write that 28 – 12 = something x 2

Consider Fig. 8. What would you do?
Ask your students "How would you represent Fig. 8 as an equation?"



What do you think about this challenge?

Fig. 8

The revisions to the Alberta curriculum in 2007 introduced the topic of equality in Grade 1. Equality was not part of the Grade One outcomes prior to this point. The intent of this article was to draw the reader's focus to the importance of the equal sign indicating a relationship. Students need to understand the equal sign thoroughly as a relationship before ever using it to write equations.

The examples to this point have only dealt with equality as a relationship between quantities: how much or how many? But equal is a relationship that applies to a comparison of any measure: equal length; equal distance around; equal weight, equivalent units as in cm➔m, etc.

Where is equality specifically identified in the Alberta Program of Studies?

Grade One:

- Describe equality as a balance and inequality as an imbalance, concretely and pictorially. (0 to 20) *[C, CN, R, V]*
- Record equalities, using the equal symbol. *[C, CN, PS, V]*

Grade Two:

- Demonstrate and explain the meaning of equality and inequality, concretely and pictorially. *[C, CN, R, V]*
- Record equalities and inequalities symbolically, using the equal symbol or the not equal symbol. *[C, CN, R, V]*

Grade Six:

- Demonstrate and explain the meaning of preservation of equality, concretely and pictorially. *[C, CN, PS, R, V]*

However, understanding equals forms the basis for success with all operations in K-12 mathematics.

During the 2015-2016 school year, the Elementary Mathematics Professional Learning Opportunities (EMPLO) website was launched. A grant from Alberta Education provided an opportunity for collaborative teams to provide access to free resources as part of a coordinated effort to ensure that K-6 education stakeholders share common understandings around the expectations of the Alberta Program of Studies for Mathematics.

During the 2016-2017 year, this website will continue to morph and grow through the addition of resources, activities, research, and evidence of learner understanding. Once you visit, check back often. If you are on Twitter, follow us at @EMPL_AB in order to receive update alerts and contest announcements.

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