

THE SQUARE-ROOT SEARCH

NUMBER • GEOMETRY • LOGIC

- Properties of squares
- Properties of rectangles
- Square roots
- Estimation

Getting Ready

What You'll Need

Base Ten Blocks, 1 set per pair

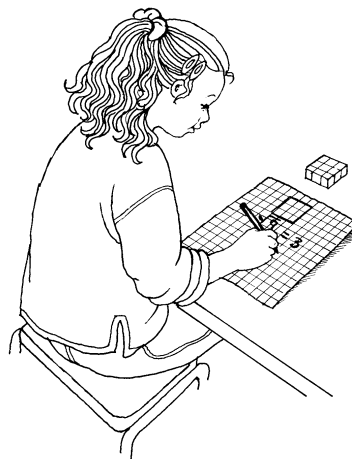
Base Ten Block Grid Paper, 1 sheet per child, page 93

Calculators with square-root keys, 1 per child (optional)

Overview

Children use Base Ten Blocks to build squares and determine the square root of the area of each. Then they investigate ways to find the square roots of nonsquare rectangles. In this activity, children have the opportunity to:

- ◆ explore the concepts of square roots and perfect squares
- ◆ approximate the square roots of imperfect squares
- ◆ become familiar with square-root notation



The Activity

Children should understand that a square is a rectangle with all four sides equal in length.

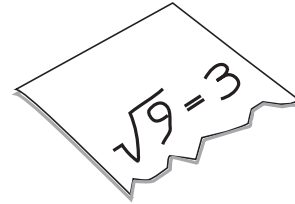
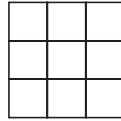
Introducing

- ◆ Distribute grid paper. Have children outline a 3×3 array on the paper and then model it with Base Ten units.
- ◆ Call on a volunteer to identify the *area* of the square array and the length of a *side*.
- ◆ Now have children outline a 5×5 array and model it with units.
- ◆ Call for someone to identify the area of this square and the length of a side.
- ◆ Explain that when a number multiplied by itself produces a product that forms a square array, the number is called a *square root*; the product is called a *square number*, or *perfect square*.
- ◆ Draw the square-root symbol on the chalkboard. Then write $\sqrt{9} = 3$. Read this aloud as, "The square root of nine equals three." (Be sure children understand that 9 is the area of the square of which 3 is a side.) Have children copy the notation below their 3×3 array.
- ◆ Ask children to write the notation for "The square root of twenty-five equals five" below their 5×5 array. Then write $\sqrt{25} = 5$ on the board so children can check their work.

On Their Own

How can you use Base 10 Blocks to search for square roots?

- Work with a partner. Build a square using any kind of Base 10 Block.
- Find the number of units on 1 side of your square. Then find the area. The area is a square number. The measure of a side is the square root of the square number.
- Record your square. Then write its area as a square number and its equivalent square root this way:
- Build 10 different squares. Record each area as a square number with its square root.
- Now, build a rectangle that is not a square!
- What is the area of your rectangle? Talk about how you might search for the square root of its area.
- Build a few more nonsquare rectangles.
- Search for the square roots of your rectangles. Try to complete sentences like these about each rectangle:



“The square root of the area of this rectangle is about ____.”

“The square root of the area is greater than ____, but less than ____.”

The Bigger Picture

Thinking and Sharing

Use some of children’s findings to create a chart of the square numbers from 1 to 100 and their corresponding square roots. Then lead children to discuss how they tried to find the square roots of nonsquare rectangles.

Use prompts like these to promote class discussion:

- ◆ What are the areas of some of the squares you built? What did you do to find the square root of each area?
- ◆ What patterns do you see in the chart of square numbers and square roots?
- ◆ How can you find the square root of any square number?
- ◆ How did finding the square roots of square numbers help you to search for the square roots of rectangular numbers?
- ◆ How did you find the square root of a rectangular number? How did you record your work?

Extending the Activity

1. Identify the square-root key on children's calculators. Demonstrate how to confirm some of the square roots that children have already found by keying in a square number and then pressing the square-root key. Then

Teacher Talk

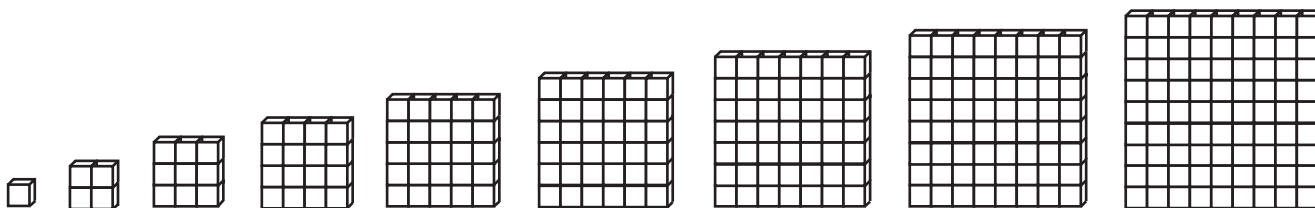
Where's the Mathematics?

Children who have had experience building arrays will have little difficulty in using Base Ten Blocks to build squares and then determining their square roots. Most children will soon see that their squares model square numbers and that additional square numbers can be produced by multiplying any whole number by itself.

By scrutinizing an ordered list of the perfect squares from 1 to 100 and their respective square roots, children are likely to notice patterns. They may point out that the square numbers are alternately odd and even and that they seem to “grow faster” than do their square roots. When children notice that the square roots replicate the order of the counting numbers, they will realize that the next square root is 11 and that multiplying 11 by itself will yield the next square number, 121.

| | | | | | | | | | | |
|----------------|---|---|---|----|----|----|----|----|----|-----|
| Square Numbers | 1 | 4 | 9 | 16 | 25 | 36 | 49 | 64 | 81 | 100 |
| Square Roots | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |

Base Ten Blocks are ideal for modeling perfect squares and their square roots. The first 9 perfect squares—those with areas of from 1 cm^2 to 81 cm^2 —can be modeled with *units alone* this way:



The perfect squares from 121 to 361 can be modeled with various *combinations of flats, longs, and units*. Squares with areas such as 100 cm^2 , 400 cm^2 , and 900 cm^2 can most easily be modeled with *flats alone*.

The challenging part of this activity presents itself when children begin their search for the square roots of their rectangular regions. Most will see that finding the area of each rectangle would be a good place to begin. They may do this either by multiplying length by width or by counting the units that make up a rectangle.

Children may start by building “almost square” rectangles, realizing that it might be easy to apply to these what they learned from working with squares. One pair, for example, had just built a 16×16 square and determined that its area was 256 square units. They added units to that square to

list various “imperfect” squares and allow children to use their calculators to find the square roots of these numbers.

2. Have children use Base Ten Blocks to find all the perfect squares between 100 and 400 along with their respective square roots.

model a 17×17 square, then found its area to be 289 square units. Now, the children removed one row of units from their square model to show a 16×17 rectangle. As soon as they found the area of the rectangle, 272, one member of the pair shouted, “We know that the perfect square 256 has a square root of 16 and the perfect square 289 has a square root of 17. Since this imperfect square, 272, comes between 256 and 289, its square root has to be somewhere between 16 and 17!” It took a while for her to convince her partner of this, but finally the pair wrote these sentences:

“Our rectangle has an area of 272 and the square root of 272 has to be a little more than 16.”

“The square root of 272 is more than 16, but less than 17.”

After building an elongated rectangle, some children will determine area and then find a reasonable approximation of the square root based on area alone. Others who start out in the same way may be unsure of what to do after finding area. Such children may ultimately decide to rearrange the blocks in their rectangle to make it more compact, or “squarish.” They may then identify the largest square contained in the new rectangle and estimate the square root based on it. For example, after building a 10×2 rectangle, one pair rearranged the blocks to form a 5×4 rectangle. Then they said, “The biggest square inside this rectangle is 16 square units, which has a square root of 4. Since our rectangle is bigger than this square, we know that our rectangle’s square root has to be more than 4.” The pair then built the next larger square, a 5×5 , and reasoned, “The next bigger square is 25. It has a square root of 5. Now we’re sure that the square root of the area of our rectangle is a number between 4 and 5.”



10 x 2 Rectangle



5 x 4 Rectangle
(4 x 4 Square)



5 x 5 Square

Children will conclude that square numbers have whole-number square roots. As they approximate the square roots of rectangular numbers, some children will also conclude that the square roots of these numbers cannot be whole numbers and so must be fractions or decimals. (Others may decide that rectangular numbers have no square roots at all!)