

Lesson 2 – Using spaghetti to discuss the Law of Sines

Objective: This lesson will help students gain a better understanding of the law of sines. In particular, this will help students understand the ambiguous case and see why it is ambiguous.

Materials Needed:

Overhead projector
Spaghetti
Protractor

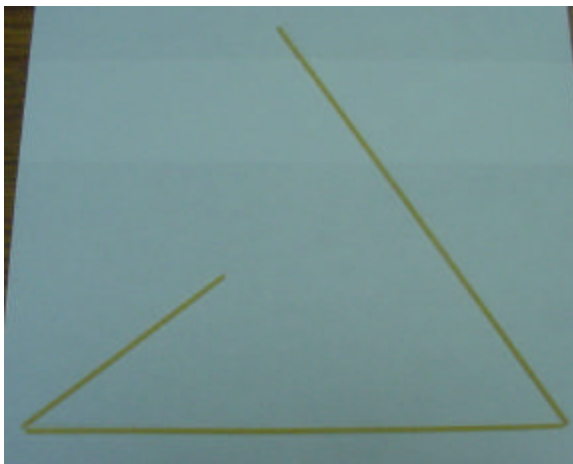
Procedure:

When discussing the Law of Sines, students sometimes have difficulty understanding why this method works. This lesson gives a visual image that can be manipulated so the students can see what is happening.

There are three cases that need to be discussed when covering the Law of Sines, AAS, ASA, and SSA. We will take these in that order. The first two are relatively straightforward and will not require a great deal of explanation. I have found that if you place the spaghetti illustrations on the overhead it is easy to manipulate and all of the students can easily see what is happening.

AAS

To illustrate AAS you will need two full noodles of spaghetti and one shorter noodle. The given information in this case is two angles and a non-included side. The shorter noodle of spaghetti will serve as our given side.



Place a full length of spaghetti horizontally on the overhead. With the second full noodle, create an acute angle with the horizontal noodle. You may wish to have a student give you an angle measurement less than 180° , preferably around 60° just to make the illustration easier to see. You can make the correct angle using the protractor. Have another student give you a second angle measurement less than 180° - the first angle. Create this angle with the shorter noodle. I recommend that you originally

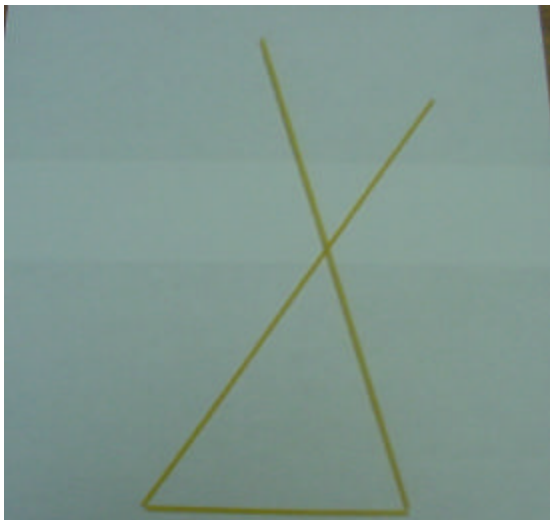
create the angle at the opposite end of the horizontal noodle. This will originally cause some confusion because it will likely not form a triangle. Ask the students if this means

that no triangle can be created with those measurements. After you have given them time for discussion, remind them that the horizontal noodle does not represent a given side. Therefore we can change the length of that side. Maintaining the given angle measurement, slide the shorter noodle across the horizontal noodle until a triangle is formed. Illustrate to the students that the original angle chosen could have been acute or obtuse and the triangle would still be created. Also note to the students that if we are all given the same pieces of information, then we will all end with congruent triangles.

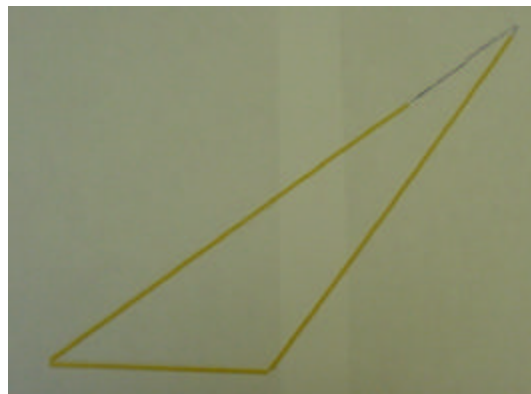


ASA

You can use the same noodles of spaghetti to illustrate this case. However, remind the students that we are now given an included side. To illustrate this, I recommend placing the shorter noodle horizontally. Since this side length is now given, the two angles must be created at the endpoints of the noodle. Again ask the students to give two angle measurements whose sum is less than 180° . I suggest you work this illustration twice. Once with two acute angles and once with an obtuse angle.



For the case with two acute angles, use the protractor to create the correct angle measurements. The students will see that the two unknown sides will clearly intersect at a point, forming the triangle. Remind the students that any access length is not a problem since the lengths of those two sides are not given.



For the case with an obtuse angle, the students may claim that they cannot make the triangle, however, remind them that the

side lengths are not given and they can extend those sides as long as necessary to form the triangle. Again, point out to the students that in either case, if we are given the same information, we will obtain congruent triangles.

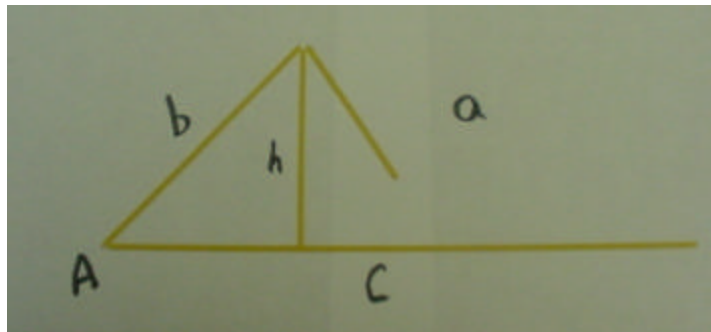
SSA

Not only is this illustration excellent for demonstrating the ambiguous case, it will convince students why we do not have a SSA congruency theorem.

For this illustration you will need four noodles of spaghetti, one full length and three shorter noodles of varying lengths. You will likely want to break these pieces off as you go since often the length you need depends on the angle the students choose. This case has multiple sub-cases, we will go through an illustration for each of them.

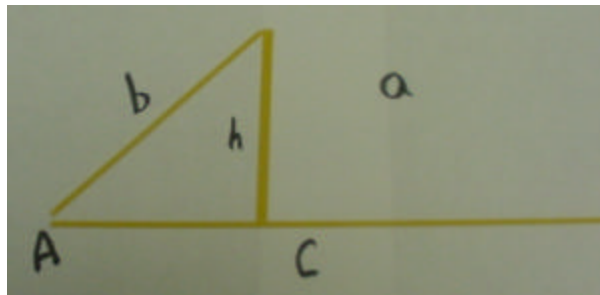
Case 1 – given angle A is acute and $a < h$ (side opposite angle a is less than the height of the triangle.)

Note that in this case, nothing is said about how b relates to a and h . Thus we can choose b to be any length we want. Break off a noodle at any length and place it at the left endpoint of the longer noodle. The angle formed here is the given angle (chosen by the students if desired) so you will need to use the protractor to measure the angle. Since b is a given side length we will break off another noodle and place it at the opposite endpoint of b and perpendicular to the unknown side. This represents the height, h . We have now restricted our options for a . We know that its length must be less than h . Break off a noodle to represent a that will be shorter than h . Show the students that there is no where to place a so that a triangle is formed.

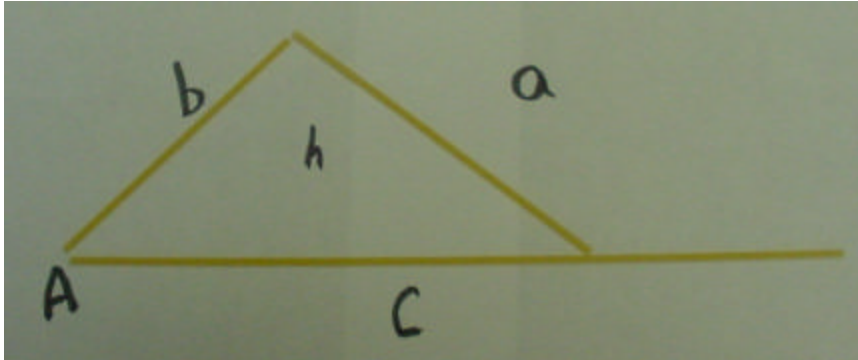


Case 2 – given angle A is acute and $a = h$.

This case is very similar to the previous case. The only change that will need to be made is to break off a noodle so that a is the same length as h rather than being shorter than h . The students should see that there is only one triangle that can be formed and that is a right triangle.



Case 3 – given angle A is acute with $a > b$.

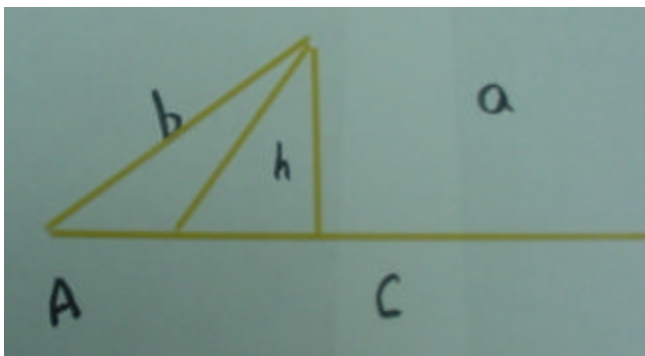
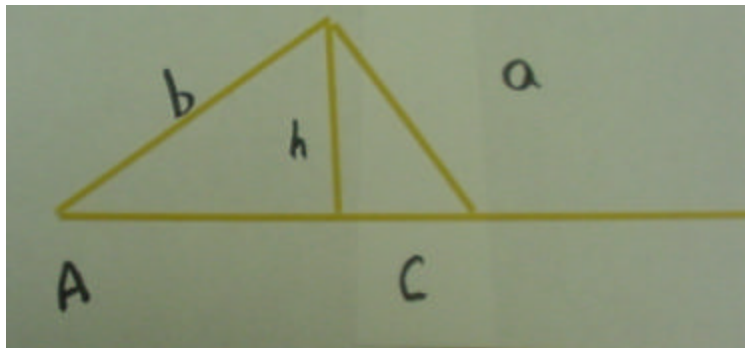


This case may be the easiest to illustrate. Since the height does not matter, simply choose two noodles of different lengths. Choose the shorter of the two to represent b and

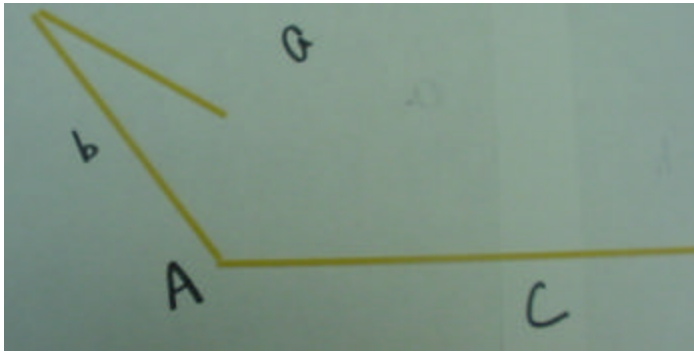
make the appropriate angle A. It will be easy for the students to see that there is only one position for a that will make a triangle. That is the correct position.

Case 4 – given angle A is acute with $h < a < b$

In this case you will have to be more careful. The easiest way to illustrate this is to choose three noodles of spaghetti of varying lengths. The longest of these noodles is to be side b . The shortest will be the height. I recommend that you do not choose a particular angle for this case. Create a right triangle with h , b and the unknown side. The angle between b and c is now set and cannot be changed. This will be easier than trying to break spaghetti off at the needed points.



This case could result in two triangles. Demonstrate to the students that side a could be placed on either side of the height to form a triangle. This is also an excellent time to discuss why we do not have a SSA congruency theorem.



Case 5 – given angle A is obtuse with $a \leq b$

In this case you will need to create an obtuse angle A and illustrate that if $a \leq b$ we will not be able to form a triangle.

Case 6 – given angle A is obtuse with $b < a$

For this final case you can use the same noodles that were used in the previous example. By switching a and b the students will see that one and only one triangle can be formed.

