## Transformations

Lesson 2

Transformation - a change made to a figure or a relation such that the figure or graph of the relation is shifted or changed in shape.




A parent function (in black) can undergo any number of transformations as shown in the above diagram. In this unit we will explore how functions are transformed and how that relates to the graph of the transformed function.

## Translations

Translation - shifts the figure or graph of the relation up or down or left or right.

## Vertical Translation

We will compare the graphs of $y=f(x)$ and $y-k=f(x)$ or $y=f(x)+k$ the next page.

Example 1: Consider the function $f(x)=|x|$.
a. Make a table of values for the equations and then sketch the graphs.

Use your calculator by putting the equation into $y=$ and then looking at the table or using trace
$y=|x| \quad y-3=|x| \quad y=|x|-1$


Use the input values $-3,-2,-1,0,1,2,3$


## Conclusion

## For a vertical translation:

The value of ' $k$ ' vertically shifts the graph up or down, depending on its value.


## Horizontal Translation

We will compare the Graphs of

$$
y=f(x) \quad \text { and } \quad y=f(x-h)
$$

Example 2: Consider the function $f(x)=|x|$
a. Make a table of values for the equations and then sketch the graphs.

$$
y=|x| \quad y=|x+2| \quad y=|x-3|
$$



Use the input values $-3,-2,-1,0,1,2,3$


## Conclusion

## For a Horizontal Translation

The value of ' $h$ ' horizontally shifts the graph left or right, depending on its value.


Example 3: Sketch each of the following functions.
Rewrite each of the equations in terms of $f$

$$
\begin{aligned}
& y=x^{2} \\
& y=(x-3)^{2}
\end{aligned}
$$

$$
\begin{aligned}
& y=x^{3} \\
& y=(x+4)^{3}
\end{aligned}
$$




Sketch each of the following functions.
Rewrite each of the equations in the form $y=f(x-h)$
$y=\frac{1}{x}$
$y=\frac{1}{(x-2)}$


$$
\begin{aligned}
& y=\sqrt{x} \\
& y=\sqrt{(x-2)}
\end{aligned}
$$



Example 4: Sketch each of the following functions.
Rewrite each of the equations in terms of $f$

$$
\begin{aligned}
& y=x^{2} \\
& y=x^{2}-3
\end{aligned}
$$

$$
\begin{aligned}
& y=x^{3} \\
& y=x^{3}+2
\end{aligned}
$$




Sketch each of the following functions.
Rewrite each of the equations in the form $y-k=f(x)$ or $y=f(x)+k$ $y=\frac{1}{x}$

$$
y=\sqrt{x}
$$

$y=\frac{1}{x}+3$

$$
y=\sqrt{x}-3
$$



Describe how the graphs of $y=f(x)+3$ and $y=f(x)-3$ compare to the graph of $y=f(x)$ ?

Relative to the graph of $y=f(x)$, what information about the graph of $y=f(x)+k$ does ' $k$ ' provide?

Describe how the graphs of $y=f(x+3)$ and $y=f(x-3)$ compare to the graph of $y=f(x)$ ?

Relative to the graph of $y=f(x)$, what information about the graph of $y=f(x-h)$ does ' $h$ ' provide?

How is the graph of a function $y=f(x)$ related to the graph of $y=f(x)+k$ when $k>0$ or $k<0$ ?

How is the graph of a function $y=f(x)$ related to the graph of $y=f(x-h)$ when $h>0$ or $h<0$ ?

Describe how the parameters $h$ and $k$ affect the properties of the graph of a function. Consider such things as shape, orientation, $x$ intercepts, $y$-intercept, domain and range.

## Combining Horizontal and Vertical Translations

Example 5: $\quad$ Sketch the graph of the function $\quad y=|x-4|+3$
a. Start with the basic function using key points.
b. Apply the horizontal translation

$$
y=|x-4|
$$

c. Apply the vertical translation

$$
y=|x-4|+3
$$

Mapping points: $\quad(x, y) \rightarrow(x+4, y+3)$


Mapping notation can be used to show the relationship of a set of points, $(x, y)$, on the original graph and a set of corresponding points on the transformed, or image, graph.

Every point on the basic (original) function has been shifted 4 units right and 3 units up. Check a few points.

Your turn: Sketch the graph of the function $y=(x+5)^{2}-2$
a. Start with the basic function using key points.
b. Apply the horizontal translation
c. Apply the vertical translation


Map the points: $(x, y) \rightarrow(x \quad, y \quad)$

## Determine the Equation of a Translated Function

Example 6: Determine the translation that has been applied to the graph of $y=f(x)$ to obtain the graph of $g(x)$
Determine the equation of the translated function in the form $y-k=f(x-h)$.

Choose key points on the original graph, $f(x)$ and locate the image points on the graph of $g(x)$

$$
\begin{array}{ccc}
(0,0) \rightarrow( & , & ) \\
(-1,1) \rightarrow( & , & ) \\
(,) \rightarrow( & , & ) \\
(,)) \rightarrow( & , & ) \\
(,)) \rightarrow( & , & ) \\
(,) \rightarrow( & , & )
\end{array}
$$



Your turn: Determine the translation that has been applied to the graph of $y=f(x)$ to obtain the graph of $g(x)$.
Determine the equation of the translated function in the form $y-k=f(x-h)$.

Choose key points on the original graph, $f(x)$ and locate the image points on the graph of $g(x)$

$$
\begin{array}{lll}
(\quad, \quad) \rightarrow( & , & ) \\
(\quad, \quad) \rightarrow( & , & ) \\
(, ~) \rightarrow( & , & ) \\
(, ~) \rightarrow( & , & ) \\
(\quad,) \rightarrow( & , & )
\end{array}
$$



Try: Determine the translation that has been applied to the graph of $y=f(x)$ to obtain the graph of $g(x)$
Determine the equation of the translated function in the form $y-k=f(x-h)$.

Choose key points on the original graph, $f(x)$ and locate the image points on the graph of $g(x)$


http://www.ronblond.com/M12/Fencd.APPLET/index.html

## ,

## In Summary:

Translations are transformations that shift all points on the graph of the function up, down, left or right without changing the shape or orientation of the graph.

## See summary page 12 in text

# Homework 

1. Assignment Handout "Translations"
2. Text Pages 12-15, Exercises \# 1,3-8, 10-12, 15, 17
(0) Translations Assignment 1.doc
