Lesson 4

Stretches of Functions

Warm-up -

a. If $g(x)$ is a reflection of $f(x)$ in the y-axis, write the equation of $g(x)$ in terms of $f(x)$.

b. What points are invariant?

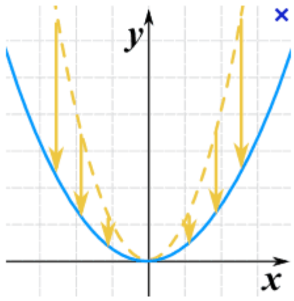
c. If $g(x)$ is a reflection of $f(x)$ in the x-axis, write the equation of $g(x)$ in terms of $f(x)$.

d. What points are invariant?

e. If $g(x)$ is a reflection of $f(x)$ in the line y=x, write the equation of $g(x)$ in terms of $f(x)$.

f. What points are invariant?

Part I: Vertical Stretches



- When a graph is vertically stretched, the x-intercepts remain constant (invariant) but the distance from the other points, to the x-axis, has been proportionally changed.

A function is vertically stretched by a factor of 'a' as shown in the function notation $y = af(x)$

The mapping of a point from its position on $y = f(x)$ to its image point on $y = af(x)$ would be,

$$(x, y) \rightarrow (x, ay)$$

If $|a| > 1$, the graph of $y = f(x)$ expands (narrows) vertically by a factor of 'a'.

If $0 < |a| < 1$, the graph of $y = f(x)$ is compressed (widens) vertically by a factor of 'a'.

If $|a| < 0$, the graph of will reflect in the x-axis as well.

Note: To find the value of 'a' , isolate 'y'

Example:

$2y = x$

$y = \frac{1}{2}x$

$a = \frac{1}{2}$

This is a vertical stretch by a factor of $\frac{1}{2}$.

Each point on the transformed graph is $\frac{1}{2}$ the distance from the x-axis compared to its original position.

$\frac{1}{3}y = x$

$y = 3x$

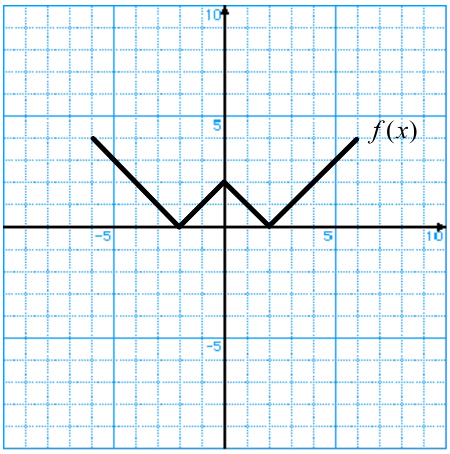
$a = 3$

This is a vertical stretch by a factor of 3. Each point on the transformed graph is 3 times the distance from the x-axis compared to its original position.

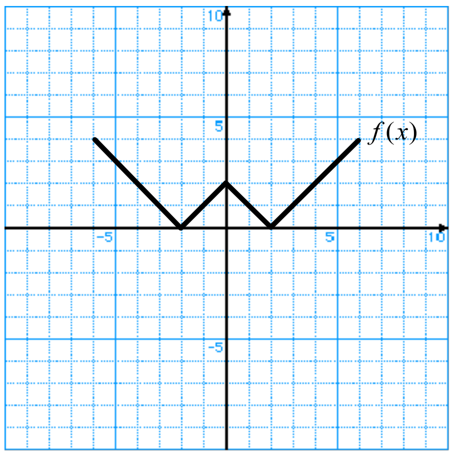
Investigate: Given the graph of $y = f(x)$,

a. transform the graph of $f(x)$ to sketch the graph of $g(x)$
b. describe the transformation.
c. state any invariant points
d. state the domain and range of the functions

$g(x) = 2f(x)$



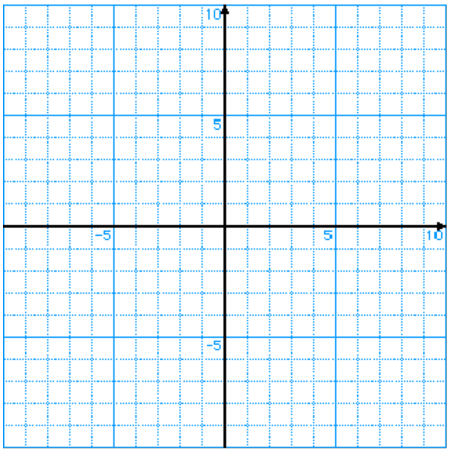
$g(x) = \frac{1}{2}f(x)$



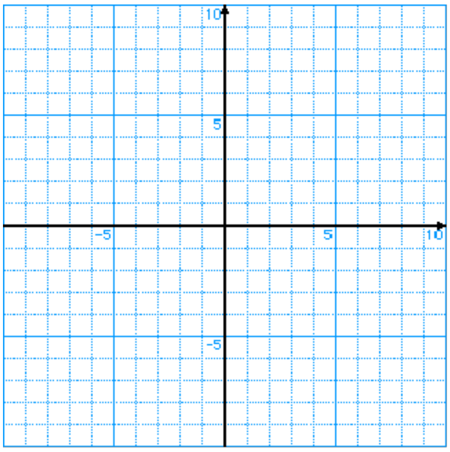
Your turn: Given the graph of $f(x) = x^2$,

a. transform the graph of $f(x)$ to sketch the graph of $g(x)$
b. describe the transformation.
c. state any invariant points
d. state the domain and range of the functions

$g(x) = 4f(x)$

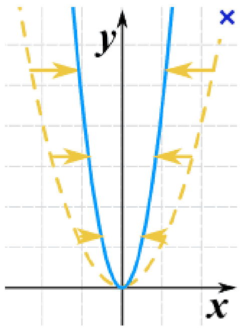


$g(x) = \frac{1}{3}f(x)$



Part II: Horizontal Stretches

- When a graph is horizontally stretched, the y-intercept remains constant (invariant) but the distance from the other points, to the y-axis, has been proportionally changed.



The mapping of a point from its position on $y = f(x)$ to its image point on $y = f(bx)$ would be,

$$(x, y) \rightarrow \left(\frac{1}{b}x, y \right)$$

If $|b| > 1$, the graph of $y = f(x)$, it is stretched (widened) horizontally by a factor of $\frac{1}{b}$.

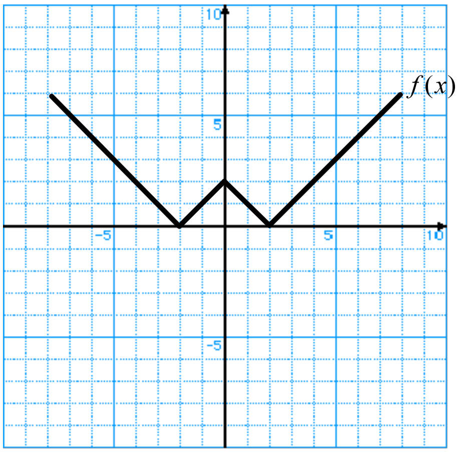
If $0 < |b| < 1$, the graph of $y = f(x)$ is stretched horizontally by a factor of $\frac{1}{b}$.

If $|b| < 0$, the graph of will reflect in the x-axis as well.

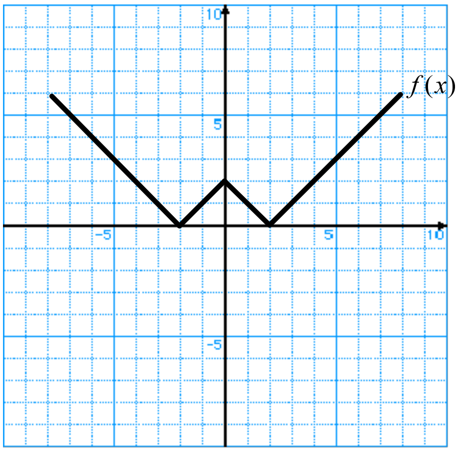
Investigate: Given the graph of $y = f(x)$,

a. transform the graph of $f(x)$ to sketch the graph of $g(x)$
b. describe the transformation.
c. state any invariant points
d. state the domain and range of the functions

$g(x) = f(2x)$



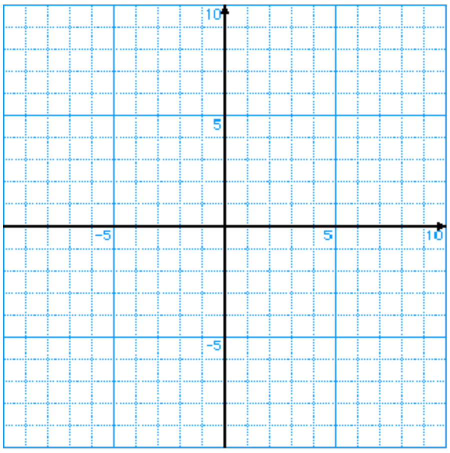
$g(x) = f\left(\frac{1}{2}x\right)$



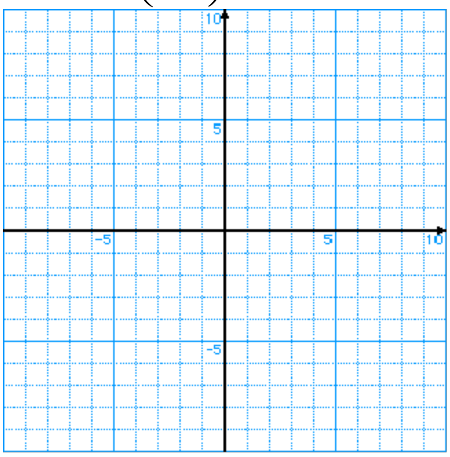
Your turn: Given the graph of $f(x) = x^2$,

a. transform the graph of $f(x)$ to sketch the graph of $g(x)$
b. describe the transformation.
c. state any invariant points
d. state the domain and range of the functions

$g(x) = 3f(x)$

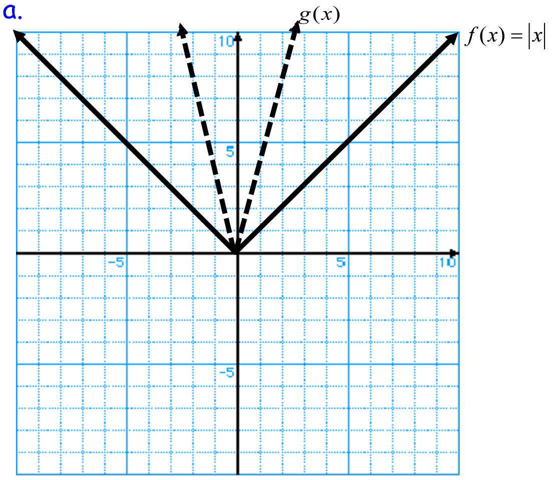


$g(x) = f\left(\frac{1}{4}x\right)$

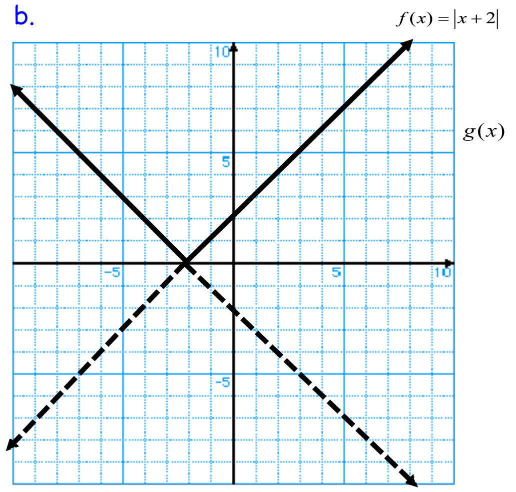


Example 3: The graph of the function $y = f(x)$ has been transformed by either a stretch or a reflection. For the following, write the equation of $g(x)$.

a.



b.

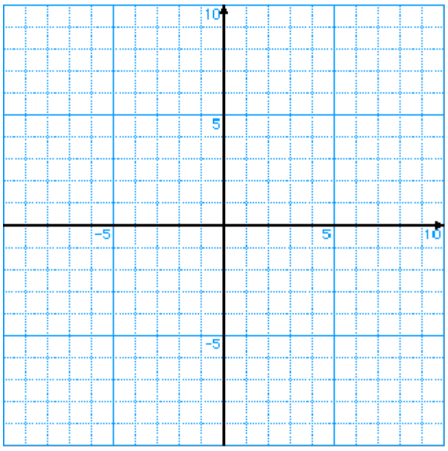


Example 4: Graph of the functions $f(x) = x^2$, $g(x) = 4x^2$ and $h(x) = (2x)^2$ on the grid below. Use a table of values from your calculator,

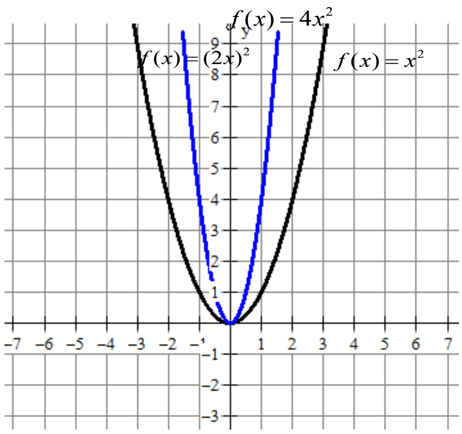
a. Explain the transformations $g(x)$ and $h(x)$ in words.

b. Write the transformed functions in terms of $f(x)$

c. What do you notice about the transformations?



Ans:



In summary;

Horizontal Stretches

- In general, for any function $y = f(x)$, the graph of a function $y = f(bx)$ where b is any real number, the resulting graph will be stretched horizontally by a factor of $\frac{1}{b}$.

- If $b < 0$, the graph is also reflected in the y-axis.

- Any points of $y = f(x)$ that lie on the y-axis are invariant under the transformation to $y = f(bx)$

Vertical Stretches

- In general, for any function $y = f(x)$, the graph of a function $y = af(x)$ where a is any real number, the resulting graph will be stretched vertically by a factor of a .

- If $a < 0$, the graph is also reflected in the x-axis.

- Any points of $y = f(x)$ that lie on the x-axis are invariant under the transformation to $y = af(x)$

Homework

1. Quiz 1: "Translations and Reflections"

2. Text Pages 28 - 31, Exercises # 2, 5, 6, 7ac, 10, 15cd, 16, C2, C4.

Transformations Quiz 1.doc

Attachments

- Reflections Assignment 1.doc
- Transformations Quiz 1.doc