Products and Quotients of Functions.

Lesson 10

Products of Functions.

We can create new functions by performing the operations of multiplication and division on other functions.

$$h(x) = f(x) \bullet g(x) \quad or \quad h(x) = (f \bullet g)(x)$$

Example 1. Determine the product of two functions. $h(x) = (f \bullet g)(x)$

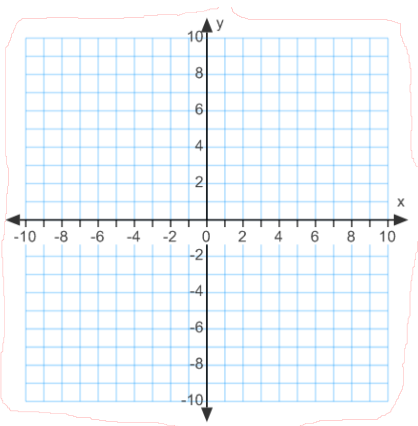
Consider the functions $f(x) = x + 2$ and $g(x) = 2x - 3$

a. Multiply $f(x)$ and $g(x)$ to determine the equation of function

$$h(x) = (f \bullet g)(x)$$
$$h(x) = f(x) \bullet g(x)$$
$$h(x) = (x+2)(2x-3)$$
$$h(x) = 2x^2 + x - 6$$

b. Sketch the graphs of $f(x), g(x)$ and $h(x)$ on the same grid.

Table of values:



x	$f(x)$	$g(x)$	$h(x)$
-2			
-1			
$-\frac{1}{4}$			
0			
1			
2			

c. State the domain and range of $h(x)$

Let's Try

Determine the product of the two functions, and the domain of the product.

Consider the functions $f(x) = x - 2$ and $g(x) = \sqrt{x-1}$

a. Multiply $f(x)$ and $g(x)$ to determine the equation of function $h(x) = (f \bullet g)(x)$

State the domain of $h(x)$

$$h(x) = (f \bullet g)(x)$$
$$h(x) = f(x) \bullet g(x)$$
$$h(x) = (x-2)\left(\sqrt{x-1}\right)$$
$$h(x) =$$

b. Sketch the graphs of $f(x), g(x)$ and $h(x)$ on the same grid.

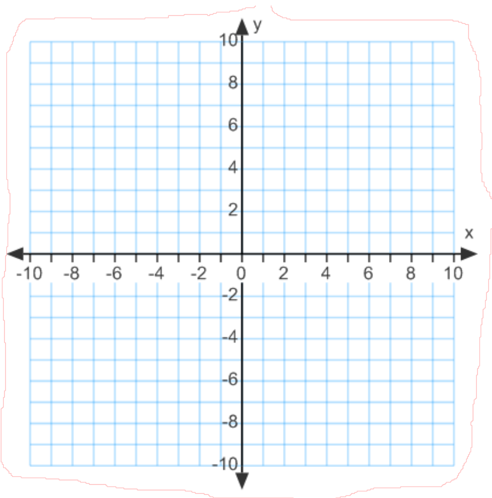


Table of values:

x	$f(x)$	$g(x)$	$h(x)$
-2			
-1			
0			
1			
2			
3			
4			
5			

c. State the domain and range of $f(x), g(x)$ and $h(x)$

d. What do you notice about the domain of the product of function relative to the domains of the original functions?

Your Turn Determine the product of the two functions.

Consider the functions $f(x)=(x+2)^2-5$ and $g(x)=3x-4$

a. Multiply $f(x)$ and $g(x)$ to determine the equation of function

$h(x)=(f \bullet g)(x)$

$h(x)=\left((x+2)^2-5\right)(3x-4)$

$h(x)=\left(x^2+4x-1\right)(3x-4)$

$h(x)=$

b. State the domain and range of $f(x), g(x)$ and $h(x)$

c. What do you notice about the domain of the product of function relative to the domains of the original functions?

Determine the Quotient of Two Functions.

Example 2: Consider the functions $f(x) = x^2 + x - 6$ and $g(x) = 2x + 6$.

a. Determine the equation of the function $h(x) = \left(\frac{g}{f}\right)(x)$

$$h(x) = \left(\frac{g}{f}\right)(x)$$

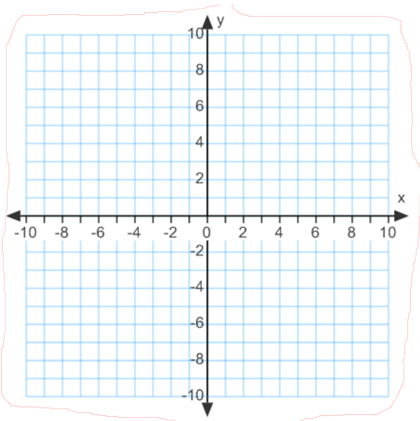
$$h(x) = \frac{g(x)}{f(x)}$$

$$h(x) = \frac{2x+6}{x^2+x-6}$$

State any restrictions on the variable

$$h(x) =$$

b. Graph the functions $f(x), g(x)$ and $h(x)$ on the same grid



x	$f(x)$	$g(x)$	$h(x)$
-2			
-1			
0			
1			
2			

Compare the domains of $f(x), g(x)$ and $h(x)$

Estimate the range of $h(x)$

Your Turn: Let $f(x)=x+2$ and $g(x)=x^2+9x+14$

a. Determine the equation of the function $h(x)=\left(\frac{f}{g}\right)(x)$

$$h(x)=\left(\frac{f}{g}\right)(x)$$

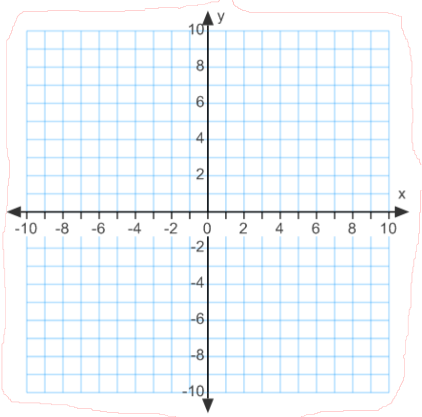
$$h(x)=\frac{f(x)}{g(x)}$$

$$h(x)=\frac{x+2}{x^2+9x+14}$$

State any restrictions on the variable

$$h(x)=$$

b. Graph the functions $f(x), g(x)$ and $h(x)$ on the same grid



x	$f(x)$	$g(x)$	$h(x)$
-2			
-1			
0			
1			
2			

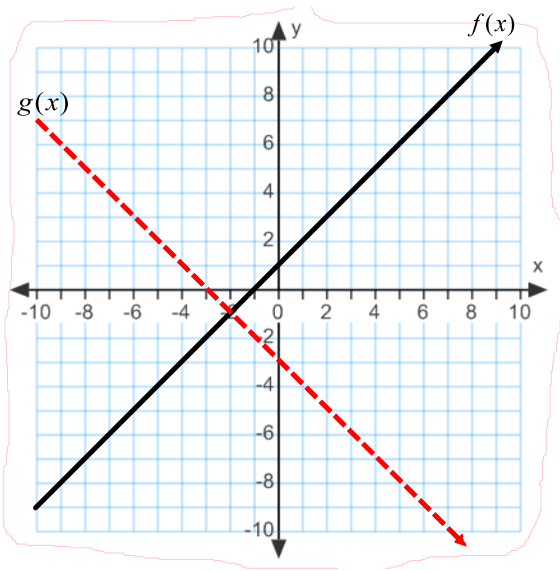
Compare the domains of $f(x), g(x)$ and $h(x)$

Estimate the range of $h(x)$

Example 3: Determine the combined function graph given the graphs of $f(x)$ and $g(x)$.

Sketch the graph of $h(x) = (f \bullet g)(x)$ given the graphs of $f(x)$ and $g(x)$.

x	$f(x)$	$g(x)$	$h(x) = (f \bullet g)(x)$
-4			
-3			
-2			
-1			
0			
1			
2			
3			
4			



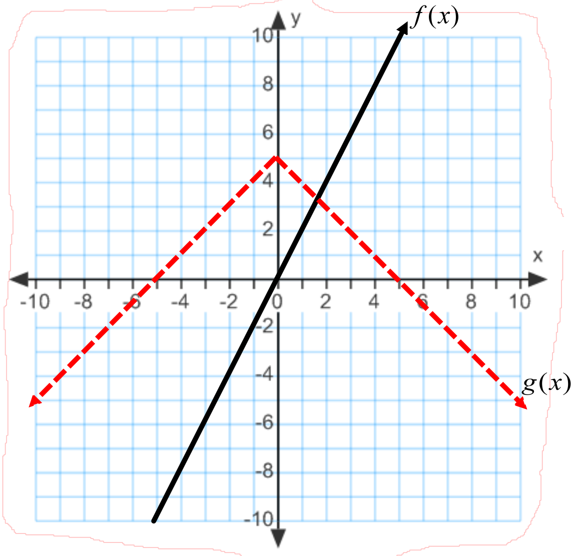
Determine the equation of $h(x)$

State the domain and range of $h(x)$

Your Turn: Determine the combined function graph given the graphs of $f(x)$ and $g(x)$.

Sketch the graph of $h(x) = (f \bullet g)(x)$ given the graphs of $f(x)$ and $g(x)$.

x	$f(x)$	$g(x)$	$h(x) = (f \bullet g)(x)$
-4			
-3			
-2			
-1			
0			
1			
2			
3			
4			



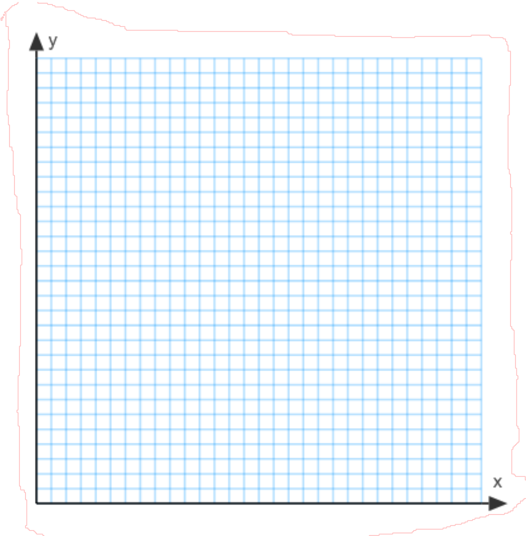
Determine the domain of $h(x)$

Analyse the graph of $h(x)$

Example 4: Modeling Using The Product of Functions (p.497, # 12)

A fish farm plans to expand. The fish population, P, in hundreds of thousands, as a function of time, t, in years, can be modeled by the function $P(t) = 6(1.03)^t$. The farm biologists use the function $F(t) = 8 + 0.04t$, where F is the amount of food, in units, that can sustain the fish population for 1 year. One unit can sustain 1 fish for 1 year.

a. Graph $P(t)$ and $F(t)$ on the same set of axes and describe the trends.



b. The amount of food per fish can be calculated using

Graph $y = \dfrac{F(t)}{P(t)}$ on a different set of axes.

Identify a suitable window setting for your graph.

c. State any restrictions on the variable. Why should these values not be considered?

d. At what time is food production per capita a maximum?

Homework

1. Text Pages 496 - 498, Exercises # 1 - 8, 11d, 16, 17, C3
2. Handout BLM 10-3: "Products and Quotients of Functions".

Attachments

Translations Assignment 1.doc