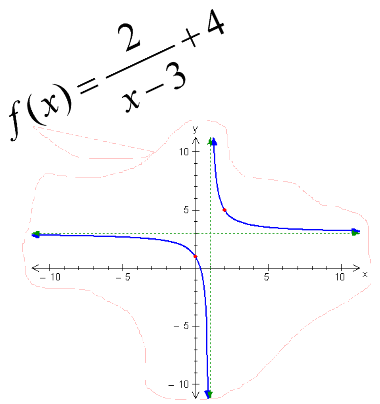
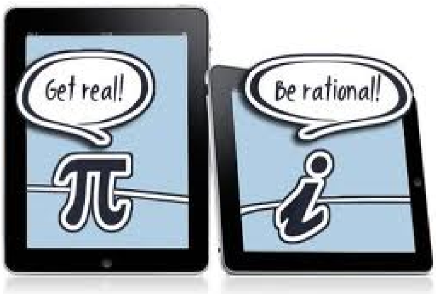
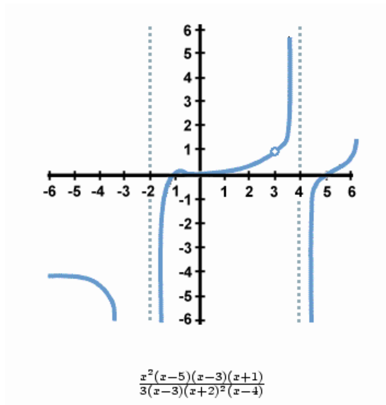
Unit 2 Part II: Rational Functions

Lesson 5





$$y=\frac{2x^2+3x+1}{x^2-5x+4}$$



Rational Function: Are functions of the form $y=\frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ polynomial expressions and $q(x)\neq 0$.

Background: We can use what we already know about the basic function $y = \frac{1}{x}$ and transformations, to write and graph equations in the form $f(x) = \frac{a}{x-h} + k$

Let's explore this. Graph $y = \frac{1}{x}$ and $f(x) = \frac{2}{x-3} + 2$ on the same grid.

Begin by stating any restrictions on the variable. **These are the non-permissible values.**
Non-permissible values may result in a vertical asymptote.
Is there a horizontal asymptote?
What are the transformations on the basic function, to obtain the function $f(x) = \frac{2}{x-3} + 2$?

a=______;____
b=______;____
h=______;____
k=______;____

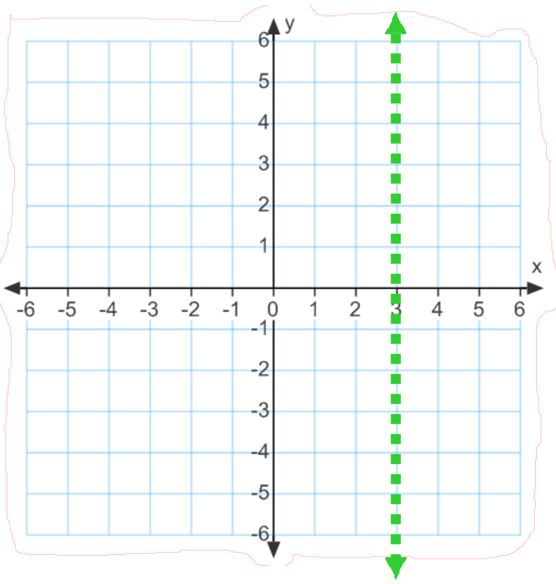
State the domain and range for each function.

$y = \frac{1}{x}$

x	y
-4	
-2	
-1	
0	npv
1	
2	
4	

$f(x) = \frac{2}{x-3} + 2$

x	$f(x)$
-1	
0	
1	
2	
3	npv
4	
5	
6	



Note: You can find the horizontal asymptote by subbing in a very large value for x.

Example 1A. Transforming rational functions.

Graph the functions $y=\frac{1}{x}$, $y=\frac{3}{x}$, $y=\frac{6}{x}$, and $y=\frac{9}{x}$ using technology.

a. Are there any non-permissible values?

b. What appears to be happening as the numerator grow larger?

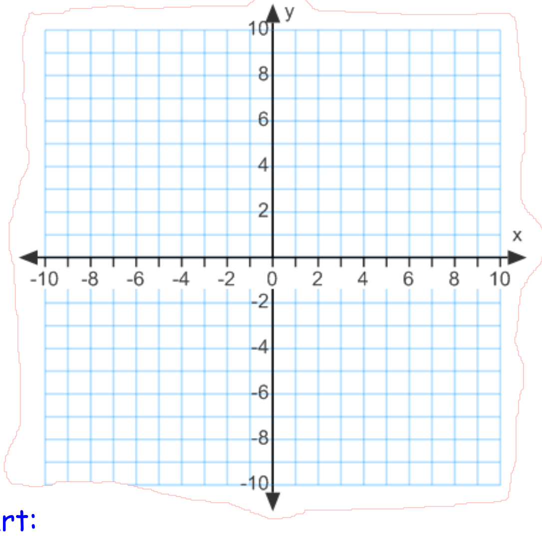
Example 1B. Consider the function $y=\frac{5}{x}$. This function can also be written in the form $y=5\left(\frac{1}{x}\right)$ so the 5 is the a value since $y=a\left(\frac{1}{x}\right)$ can be written as $y=\frac{a}{x}$

Graph the function on your calculator and fill in the following chart:

Characteristic	$y=\frac{5}{x}$
non-permissible value	
behavior near the non-permissible value	
end behavior (large values of x)	
domain	
range	
equation of the vertical asymptote	
equation of the horizontal asymptote	

Example 2: Sketch the graph of the function $f(x)=\frac{2}{x-3}+4$ using transformations, and identify any characteristics of the graph.

hint: compare $f(x)=\frac{2}{x-3}+4$ to $f(x)=\frac{a}{x-h}+k$



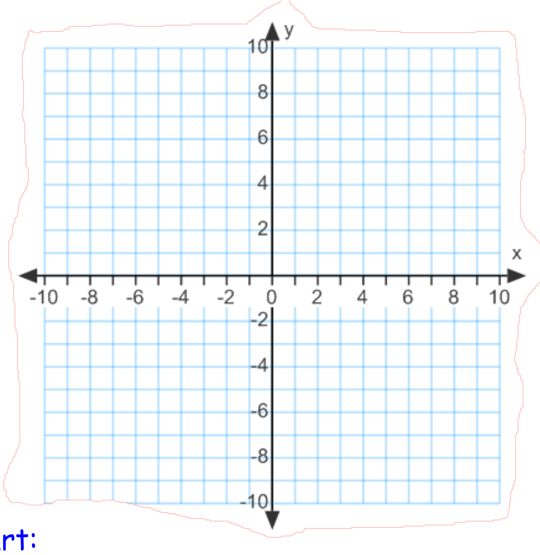
Graph the function and fill in the following chart:

Characteristic	
non-permissible value	
behavior near the non-permissible value	
end behavior (large values of x)	
domain	
range	
equation of the vertical asymptote	
equation of the horizontal asymptote	

Example 3: Sketch the graph of the function $f(x)=\dfrac{2x-1}{x-3}$

hint: we need to manipulate $f(x)=\dfrac{2x-1}{x-3}$ before

we can compare it to $f(x)=\dfrac{a}{x-h}+k$

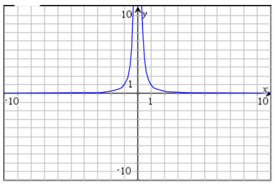


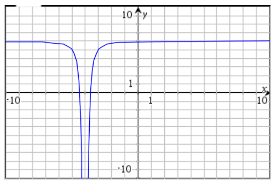
Graph the function and fill in the following chart:

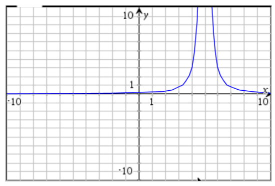
Characteristic	
non-permissible value	
behavior near the non-permissible value	
end behavior (large values of x)	
domain	
range	
equation of the vertical asymptote	
equation of the horizontal asymptote	

Comparing Rational Functions

Example 4: Consider the functions $f(x)=\frac{1}{x^2}$, $g(x)=6-\frac{1}{(x+4)^2}$, and $h(x)=\frac{3}{x^2-10x+25}$







Graph the functions and fill in the following chart:

Characteristic	$f(x)=\frac{1}{x^2}$	$g(x)=6-\frac{1}{(x+4)^2}$	$h(x)=\frac{3}{x^2-10x+25}$
non-permissible values			
behavior near the non-permissible value			
end behavior (large values of x)			
domain			
range			
equation of the vertical asymptote			
equation of the horizontal asymptote			

Your Turn: Consider the functions $f(x)=\frac{1}{x^2}$, $g(x)=\frac{-8}{(x+6)^2}$,

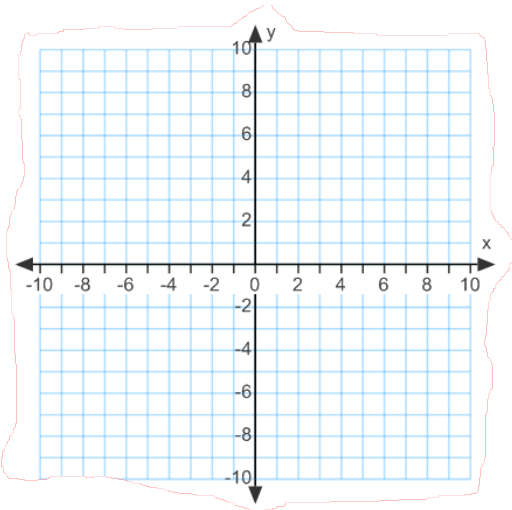
and $h(x)=\frac{4}{x^2-4x+4}-3$

Graph the functions and fill in the following chart:

Characteristic	$f(x)=\frac{1}{x^2}$	$g(x)=\frac{-8}{(x+6)^2}$	$h(x)=\frac{4}{x^2-4x+4}-3$
non-permissible values			
behavior near the non-permissible value			
end behavior (large values of x)			
domain			
range			
equation of the vertical asymptote			
equation of the horizontal asymptote			

Example 5: Determine an equation of a rational function that has an asymptote at $x=2$ and $y=4$. Explain the rationale for your equation.

Sketch the graph of your function
Identify the asymptotes



Domain:
Range:

Example 6: Describe the similarities and differences between the following graphs, without using your calculator.

$f(x) = \frac{4}{x+1} + 5$ $g(x) = 4(x+1)^2 + 5$ $h(x) = 4\sqrt{x+1} + 5$

Homework

1. Assignment Handout:

"BLM 9-2 Exploring Rational Functions and Transformations"

2. Text Pages 442 - 445, Exercises # 1 - 8, 10 - 14, 21, C2, C3

Attachments

Translations Assignment 1.doc