## Unit 3:

Exponential and Logarithmic Functions


## PartI: Exponential Functions

## Characteristics of Exponential Functions

The graph of an exponential function is a function of the form $y=c^{x}$ where $c$ is a constant, $c>0$ and $x$ is a variable.

Example 1. Investigating the graph of an exponential function.

Graph the exponential function $y=3^{x}$
Identify,
domain and range
$x$-intercept and $y$-intercept

The equation of the horizontal asymptote

| $x$ | $y$ |
| :--- | :--- |
| -3 |  |
| -2 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 2 |  |



What happens if $0<c<1 ?$

$$
y=c^{x}
$$

Graph the exponential function $\quad y=\left(\frac{1}{3}\right)^{x}$
Identify,
domain and range
$x$-intercept and
$y$-intercept

The equation of the
 horizontal asymptote

Your Turn: Graph the function $f(x)=5^{x}$

Identify the domain, range, any intercepts, whether the graph is an increasing or decreasing function and the equation of the horizontal asymptote.

| $x$ | -2 | -1 | 0 | 1 | 2 |
| :---: | :--- | :--- | :--- | :--- | :--- |
| $f(x)$ |  |  |  |  |  |



Example 2: Write the exponential functions from the given graphs.
Hint: Look for a pattern in the ordered pairs.


| $x$ | -1 | 0 | 1 | 2 |
| :---: | :--- | :--- | :--- | :--- |
| $f(x)$ |  |  |  |  |




Your Turn: What function of the form $y=c^{x}$ can be used to describe the graph below?

| $x$ | -1 | 0 | 1 | 2 |
| :---: | :---: | :---: | :---: | :---: |
| $f(x)$ |  |  |  |  |



## Example 3: Exponential Growth and Decay. Applications of the Exponential Function.

Investigate: A certain type of bacteria is doubling every 30 minutes.

| Number of <br> doubling <br> periods, n. | Number of <br> bacteria <br> present, $B$. |
| :---: | :---: |
| 0 | 1 |
| 1 | 2 |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |

1. Complete the table.
2. Write an equation that relates $B$ to $n$.
3. After 2 hours, how many bacteria will result from a single bacterium?
4. If there were 100 bacteria initially in a sample of the same type, how many bacteria would there be 2 hr later? What equation would give the number of bacteria after $n$ doubling periods, for this sample?
5. A culture has a bacterial count of 500 at the start. What equation would give the number present t hours from now?

An exponential function is a function of the form, $f(x)=a \cdot c^{x}$ where a is a constant and $\mathrm{c}>0$.
In the above function:
a would represent the initial amount of bacteria, or value or light or.......
c represents the growth or decay factor.

- If something is doubling c is equal to 2 .
- If something is growing at a rate of $7 \% \mathrm{c}$ is equal to 1.07 , that is the original amount and the amount of growth.
- If something is decaying c may be equal to 0.5
- $x$ is the amount of time something takes to double or half or grow or decay....
- $x$ is sometimes expressed as

$$
\frac{\text { time passed }}{\text { doubling time }}
$$

or....

$$
\frac{\text { time passed }}{\text { half-life }}
$$

A radioactive sample of radium ( $\mathrm{Ra}-225$ ) has a half-life of 15 days. The mass, $m$ in grams, of Ra-225 remaining over time, $t$, in 15 day intervals, can be modeled using the exponential graph shown.

1. What is the initial mass of $\mathrm{Ra}-225$ in the sample?
2. What value does the mass of $\mathrm{Ra}-225$ approach over time?
3. Write the exponential decay model that relates the mass of Ra-225 to time in 15 day intervals.

4. Write the exponential model that relates the mass of $\mathrm{Ra}-225$ to time passed.
5. Estimate how many days it would take for Ra- 225 to decay to $\frac{1}{30}$ of its original mass.

Your Turn: A certain bacteria population triples every week.
Write an exponential growth model of the form $\quad N=n(G)^{\frac{t}{p}}$
that relates the number of bacteria, N , to the time passed since the initial count, n. $G$ is the growth factor, and p is the tripling period.

1. Assignment Handouts:


BLM 7-1; Prerequisite Skills
BLM 7-1; Characteristics of Exponential Functions
2. Text Pages 342-345, Exercises \# 1-11, 13, C1, C2
(0) Translations Assignment 1.doc

