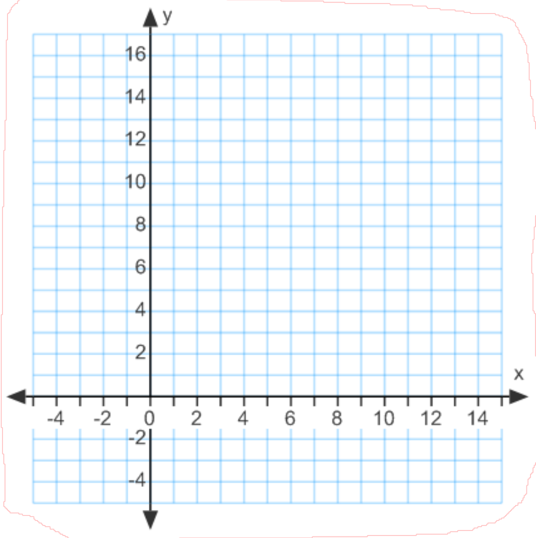
Part II: Logarithmic Functions

Lesson 5

8.1 Understanding Logarithms

Investigate: Use a table of values to graph $y=10^x$ and $y=\log x$.

x	$y=10^x$	$y=\log x$
-3		
-1		
0		
$\frac{1}{10}$		
$\frac{1}{5}$		
$\frac{1}{4}$		
$\frac{1}{2}$		
1		
2		
3		



Fill in the table below;

Function	domain	range	x-intercept	y-intercept	Equation of asymptote
$y=10^x$					
$y=\log x$					

What do you notice about these two functions?

The inverse of the function $y = c^x$ is $x = c^y$. This inverse is also a function called a logarithmic function. It is written in the form $y = \log_c x$ where c is a positive number other than 1.

exponent

$$x = c^y \qquad\qquad y = \log_c x$$

base

So a logarithm is just an exponent. Logarithms in base 10 are called common logarithms. When you write a common logarithm, you do not need to write the base. For example, we write $y = \log_{10} 3$ as $y = \log 3$.

Example 1: Evaluate the following logarithms.

a. $y = \log_7 49$ b. $y = \log_6 1$ c. $y = \log 0.001$ d. $y = \log_2 \sqrt{8}$

Your Turn: Evaluate the following logarithms.

a. $y = \log 1000000$ b. $y = \log_2 32$ c. $y = \log_9 \sqrt[5]{81}$ d. $y = \log_3 9\sqrt{3}$

Recap:

$y = c^x \quad \rightarrow$ inverse is $\rightarrow \quad x = c^y \quad \Rightarrow \quad \log_c x = y$

Important Notes:

$\log_c 1 = 0$ since in exponential form $c^0 = 1$

$\log_c c = 1$ since in exponential form $c^1 = c$

$\log_c c^x = x$ since in exponential form $c^x = c^x$

$c^{\log_c x} = x$, $x > 0$, since in logarithmic form $\log_c x = \log_c x$

Example 2: Determining the value of x in the following logarithmic forms.

(hint; express them in exponential form first)

a. $\log_5 x = -3$

b. $\log_x 36 = 2$

c. $\log_{64} x = \dfrac{2}{3}$

Try: Determine the value of x .

a. $\log_4 x = -2$

b. $\log_{16} x = -\frac{1}{4}$

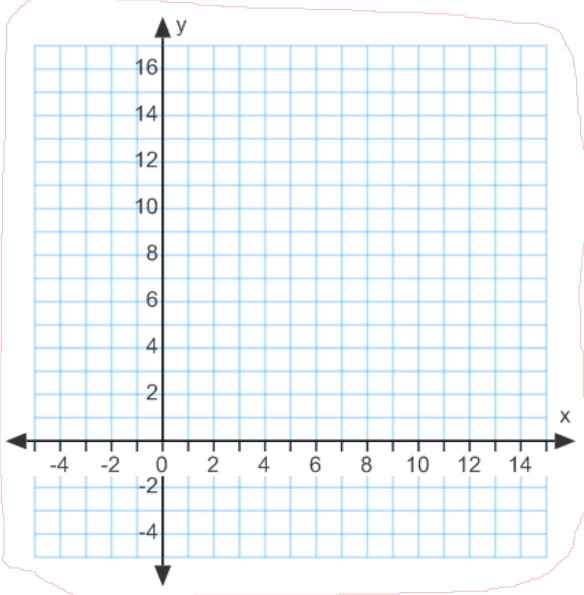
c. $\log_x 9 = \frac{2}{3}$

Example 3: Graph the Inverse of an Exponential Function.

a. State the inverse of $f(x) = 3^x$

b. Sketch the graph of the inverse. State the domain, range, any x-intercept or y-intercept and the equations of any asymptote.

$f(x) = 3^x$		$f^{-1}(x) = \log_3 x$	
x	y	x	y
-3			
-2			
-1			
0			
1			
2			
3			



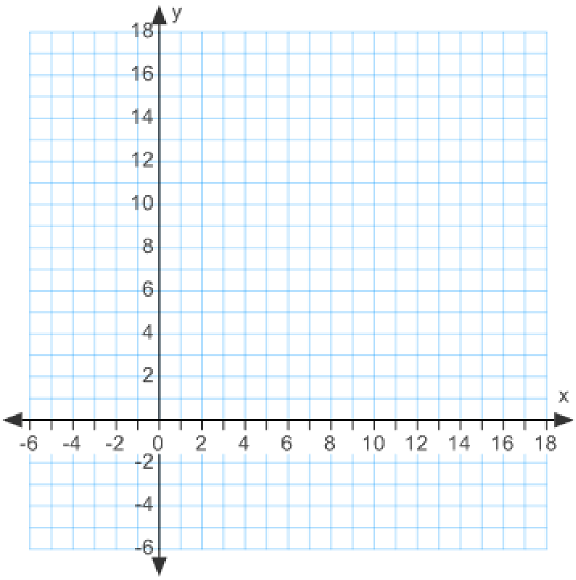
Function	domain	range	x-intercept	y-intercept	Equation of asymptote
$f(x) = 3^x$					
$f^{-1}(x) = \log_3 x$					

<u>Your Turn</u>:

a. State the inverse of $f(x) = \left(\frac{1}{2}\right)^x$

b. Sketch the graph of the inverse. State the domain, range, any x-intercept or y-intercept and the equations of any asymptote.

x	y	x	y
-3			
-2			
-1			
0			
1			
2			
3			



Function	domain	range	x-intercept	y-intercept	Equation of asymptote

Example 4: Estimating the value of a logarithm.

Without using your calculator, estimate the value of $\log_2 14$ to one decimal place.

$\log_2 14$: Think; What exponent do I apply to the base of 2, to get 14?

Example 5: Applications of Logarithms.

There are many forms of logarithmic scales such as the Richter Scale, decibel scale (loudness), and pH scales, to name a few.

The Richter Scale, developed in 1935 by Charles Richter, measures the magnitude of earthquakes.

The formula for the magnitude of an earthquake is: $M = \log \frac{A}{A_0}$

Where A is the amplitude of the ground motion, usually measured in microns, and A_0 is the amplitude for a standard earthquake. A standard earthquake has a magnitude of 0. Each increase of 1 unit on the Richter Scale is a ten-fold increase in the intensity of the earthquake.

1. In 1946, an earthquake struck Vancouver Island off the coast of British Columbia. It had an amplitude that was $10^{7.3}$ times A_0. What was the quakes magnitude on the Richter Scale?

$$M = \log\frac{A}{A_0}$$

2. The strongest recorded earthquake in Canada struck Haida Gwaii in 1949. It had a Richter magnitude of 8.1 . How many times more intense was the Haida Gwaii earthquake, compared to the standard earthquake?

$$M = \log\frac{A}{A_0}$$

3. Compare the intensity of the Haida Gwaii earthquake to that of the Vancouver Island earthquake.

$$M = \frac{A_{HG}}{A_{VI}}$$

4. The strongest measured earthquake struck Chile in 1960. It measured 9.5 on the Richter scale. How many times more intense was the Chilean earthquake than the Haida Gwaii earthquake, which measured 8.1 on the Richter scale?

$$M = \frac{A_{Chile}}{A_{HG}}$$

Homework

1. Assignment Handouts: BLM 8-1 Prerequisite Skills
   BLM 8-2 Understanding Logarithms

2. Text Pages 380 - 382, Exercises # 2 - 4, 8 - 16, 19, C1
   # 20 - 24 in groups

Attachments

Translations Assignment 1.doc