8.2 Graphing Logarithms and Transformations

Lesson 6

Explaining the effects of the parameters a, b, h, and k in $y = a\log_c\left(b\left(x-h\right)\right)+k$ on the graph of $y = \log_c x$ where $c > 1$

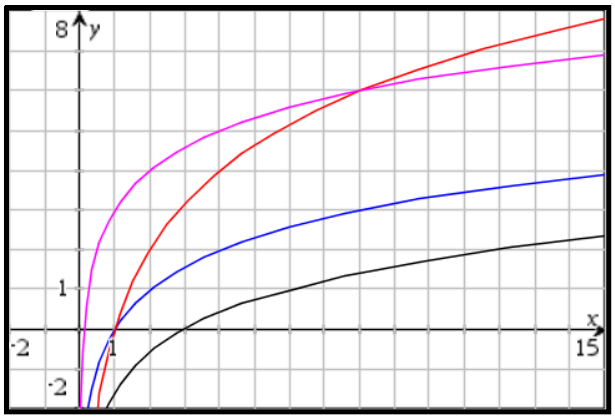
From your knowledge of transformations, predict the effects of the parameters a, b, h, and k in $y = a\log_c\left(b\left(x-h\right)\right)+k$ on the graph of the logarithmic function $y = \log_c x$, in the table below.

Parameter	Effect on the basic function graph
a	
b	
h	
k	

Only parameter ___ changes the _____ _____ and the _____ .
None of the parameters change the _____ .

The following table shows, in mapping notation, how each parameter affects the point (x, y) on the graph of $y = \log_c x$

Parameter	Transformation
a	$(x, y) \to (x, ay)$
b	$(x, y) \to \left(\frac{1}{b}x, y\right)$
h	$(x, y) \to (x+h, y)$
k	$(x, y) \to (x, y+k)$



If the blue graph is $y = \log_2 x$

Can you write the equations for the red graph:

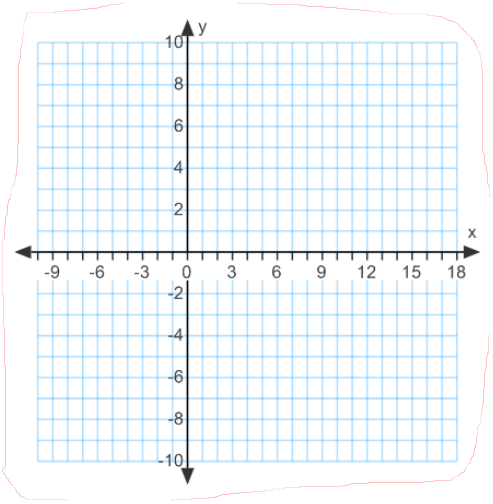
pink graph:

black graph:

Where would you sketch the graph $y = \log_2(x-4)$?

Example 1:

a. Sketch the graphs of $y = \log_4 x$ and $y = \log_4 (x+4) - 5$ on the same grid.



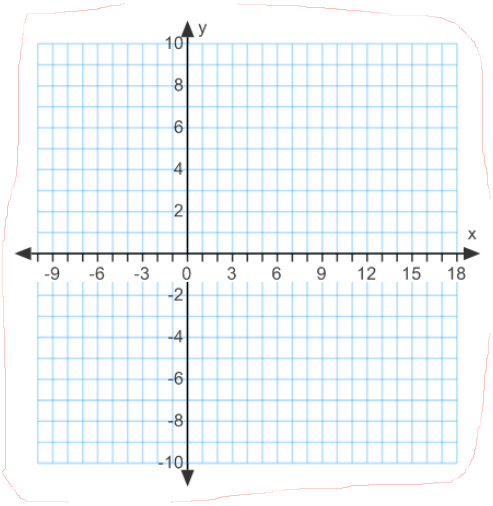
$(x, y) \rightarrow (\quad , \quad)$

$y = \log_4 x$	$y = \log_4 (x+4) - 5$
$(\quad , \quad)$	$(\quad , \quad)$

b. State the domain, range x-intercept, y-intercept, and the equation of the asymptote.

Your turn:

a. Sketch the graphs of $y = \log_3 x$ and $y = \log_3 (x+9) + 2$ on the same grid.



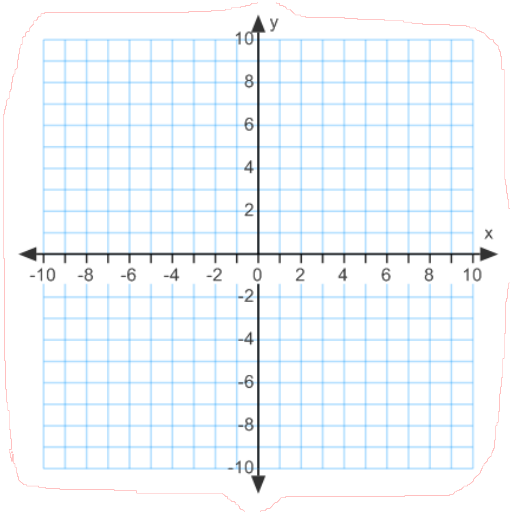
$(x, y) \rightarrow (\quad , \quad)$

$y = \log_3 x$	$y = \log_3 (x+9) + 2$
(,)	(,)

b. State the domain, range x-intercept, y-intercept, and the equation of the asymptote.

Example 2: Reflections, Stretches and Translations of Logarithmic Functions.

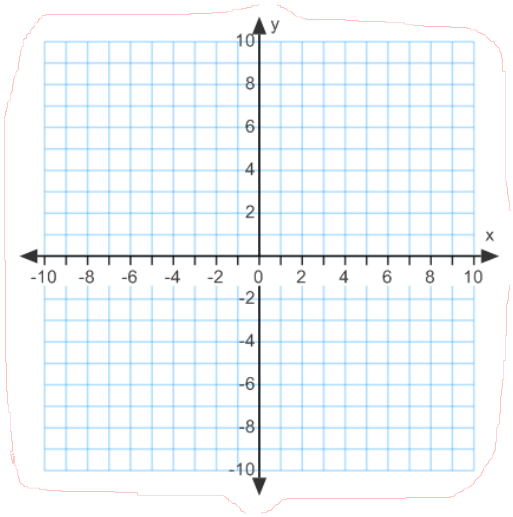
a. Use transformations to sketch the graph of $y = -\log_2(2x+6)$



$y = \log_2 x$	
(,)	(,)

b. Identify the equation of the asymptote, the domain and range, and any intercepts, if they exist.

Your Turn: a. Use transformations to sketch the graph of $y = 2\log_3(-x+1)$

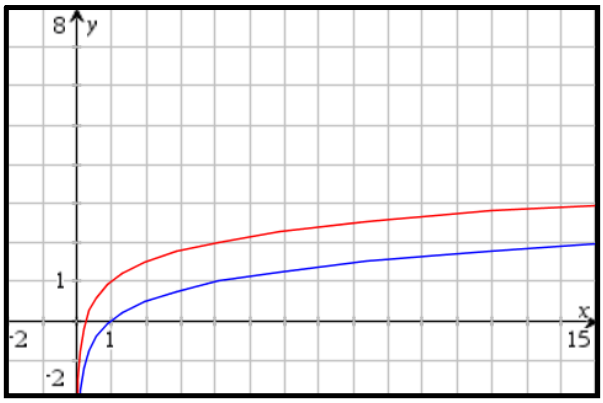


(,)	(,)

b. Identify the equation of the asymptote, the domain and range, and any intercepts, if they exist.

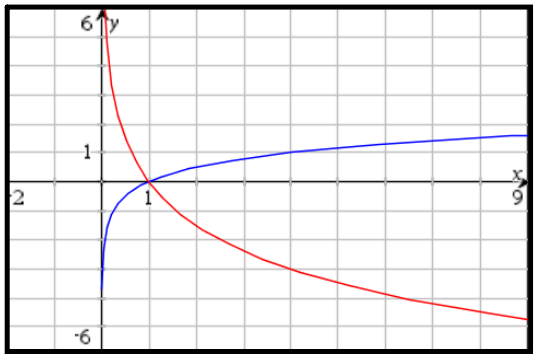
Example 3: Determine the Equation of a Logarithmic Function Given Its Graph

The red graph can be generated by stretching the blue graph of $y = \log_4 x$.
Write the equation that describes the red graph.



Try this one:

The red graph can be generated by stretching and reflecting the blue graph of $y = \log_4 x$. Write the equation that describes the red graph.



Example 3: Applications of Logarithmic Functions

a. The pH of a solution is given by the formula $pH = -\log\left[H^+\right]$

where $\left[H^+\right]$ is the solution's hydrogen ion concentration (in moles per litre).

Find the pH of the solution.

i. lemon juice: $\left[H^+\right] = 1\times10^{-2.4}$ moles per litre

ii. vinegar: $\left[H^+\right] = 1\times10^{-3}$ moles per litre

iii. orange juice: $\left[H^+\right] = 1\times10^{-3.5}$ moles per litre

b. The Richter scale, used for measuring the magnitude, R, of an earthquake, is given by the model

$$R = 0.67\log(0.37E) + 1.46$$

where E is the energy (in kilowatt-hours) released by the earthquake.

Suppose an earthquake releases 15 500 000 000 kilowatt-hours of energy. What is the earthquakes magnitude?

c. Most tornadoes last less than one hour and travel less than 20 miles. The wind speed s (in miles per hour) near the center of the tornado is related to the distance d (in miles) the tornado travels, by this model:

$$s = 93\log d + 65$$

On March 18, 1925, a tornado whose wind speed was about 280 mph struck the Midwest United States. Estimate how far the tornado traveled.

Your Turn:

a. Welders wear helmets fitted with a filter shade to protect their eyes from the intense light. The filter shade number, N is defined by the function

$$N = \frac{7(-\log T)}{3} + 1$$

where T is the fraction of visible light that passes through the filter.

A shade number of 12 is suggested for arc welding. What fraction of visible light is passed through the filter to the welder, as a percent to the nearest ten thousandth?

b. There is a logarithmic relationship between butterflies and flowers. In one study, scientists found that the relationship between the number F, of flower species that a butterfly feeds on and the number B, of butterflies observed can be modeled by the function

$$F = -2.641 + 8.958 \log B$$

Predict the number of butterfly observations in a region with 25 flower species.

Homework

1. Assignment Handout:
   BLM "Section 8.2 Transforming Logarithmic Functions"

2. Text Pages 389 - 391, Exercises # 1,4 - 14, 16, 17, C1, C3

Attachments

Translations Assignment 1.doc