Lesson 7

# 8.3 Laws of Logarithms

Investigate: Show that $\log(1000 \times 100) \neq (\log 1000)(\log 100)$

a. Use a calculator to find the approximate value of each expression. State your answer to four decimal places.

i. $\log 6 + \log 5$
ii. $\log 30$
iii. $\log 11 + \log 9$
iv. $\log 99$
v. $\log 7 + \log 3$
vi. $\log 21$

b. Based on the results in part a., suggest a possible law for $\log M + \log N$ where $M$ and $N$ are positive real numbers.

c. Use your conjecture from part b. to express $\log 1000 + \log 100$ as a single logarithm.

Show that $\log\frac{1000}{100} \neq \frac{\log 1000}{\log 100}$

a. Use a calculator to find the approximate value of each expression. State your answer to four decimal places.

i. $\log 48 - \log 4$
ii. $\log 12$
iii. $\log 35 - \log 5$
iv. $\log 7$
v. $\log 72 - \log 2$
vi. $\log 36$

b. Based on the results in part a., suggest a possible law for $\log M - \log N$ where $M$ and $N$ are positive real numbers.

c. Use your conjecture from part b. to express $\log 1000 - \log 100$ as a single logarithm.

Show that $\log 1000^2 \neq (\log 1000)^2$

a. Use a calculator to find the approximate value of each expression. State your answer to four decimal places.

i. $3\log 5$
ii. $\log 125$
iii. $4\log 2$
iv. $\log 16$
v. $2\log 7$
vi. $\log 49$

b. Based on the results in part a., suggest a possible law for $P\log M$ where $M$ is a positive real number, and $P$ is any real number.

c. Use your conjecture from part b. to express $2\log 1000$ as a logarithm without a coefficient.

Laws of Logarithms:

Product Law of Logarithms: $\log_c MN = \log_c M + \log_c N$

Quotient Law of Logarithms: $\log_c \frac{M}{N} = \log_c M - \log_c N$

Power Law of Logarithms: $\log_c M^P = P \log_c M$

These laws are true for a logarithm with any base that is a positive real number other than 1. Without a calculator, evaluate each of the following.

a. $\log_6 18 + \log_6 2$

b. $\log_2 40 - \log_2 5$

c. $4 \log_9 3$

Example 1: Write each expression in terms of individual logarithms of x, y, and z.

a. $\log_5 \frac{xy}{z}$

b. $\log_7 \sqrt[3]{x}$

c. $\log_6 \frac{1}{x^2}$

d. $\log \frac{x^3}{y\sqrt{z}}$

Your Turn: Write each expression in terms of individual logarithms of x, y, and z.

a. $\log_6 \frac{x}{y}$

b. $\log_5 \sqrt{xy}$

c. $\log_3 \frac{9}{\sqrt[3]{x^2}}$

d. $\log \frac{x^5 y}{\sqrt{z}}$

Example 2: Use the laws of logarithms to simplify and evaluate each expression.

a. $\log_6 8 + \log_6 9 - \log_6 2$

b. $\log_7 7\sqrt{7}$

c. $2\log_2 12 - \left( \log_2 6 + \frac{1}{3}\log_2 27 \right)$

Your Turn: Use the laws of logarithms to simplify and evaluate each expression.

a. $\log_5 1000 - \log_5 4 - \log_5 2$

b. $\log_3 9\sqrt{3}$

c. $2\log_3 6 - \frac{1}{2}\log_3 64 + \log_3 2$

Example 3: Write each expression as a single logarithm in simplest form. State the restrictions on the variable.

a. $\log_7 x^2 + \log_7 x - \dfrac{5\log_7 x}{2}$

b. $\log_5 (2x-2) - \log_5 (x^2+2x-3)$

Your Turn: Write each expression as a single logarithm in simplest form. State the restrictions on the variable.

a. $4\log_3 x - \frac{1}{2}(\log_3 x + 5\log_3 x)$

b. $\log_2\left(x^2 - 9\right) - \log_2\left(x^2 - x - 6\right)$

Application:

Recall that the $pH$ of a solution is defined as $pH = -\log\left[H^+\right]$

Where is the hydrogen ion concentration in moles per litre $\left(mol/L\right)$

A common ingredient in cola drinks is phosphoric acid, the same ingredient in many rust removers. A cola drink has a pH of 2.5. Milk has a pH of 6.6. How many times as acidic as milk is a cola drink?

An apple is 5 times as acidic as a pear. If a pear has a pH of 3.8, then what is the pH of the apple?

# Homework

1. Assignment Handout

"BLM Section 8.3 Laws of Logarithms"

2. Text Pages 400 - 403, Exercises # 1 - 3, 5 - 17, C1

## Attachments

Translations Assignment 1.doc