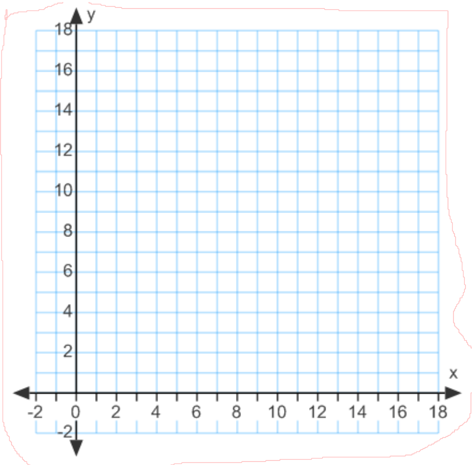
Review of Exponential and Logarithmic Functions

Lesson 9

Understanding Logarithms:

a. Sketch the graphs of $y = 2^x$ and $y = \log_2 x$ on the same grid. State the domain, range, any intercepts and the equation of the asymptotes for both functions.



$y = 2^x$	$y = \log_2 x$

Write each expression in logarithmic form.

$4^3 = 64$

$2^9 = 512$

$10^{-4} = 0.0001$

$6^x = 216$

Write each expression in exponential form.

$\log_4 256 = 4$

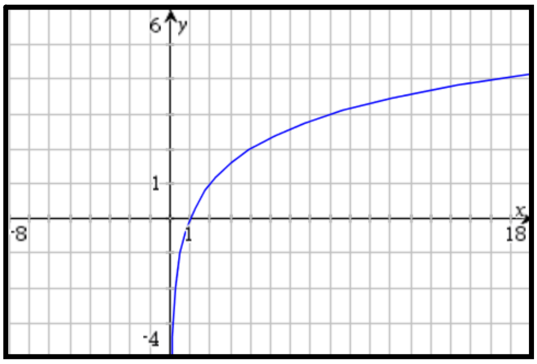
$\log_9 3 = \frac{1}{2}$

$\log 1 = 0$

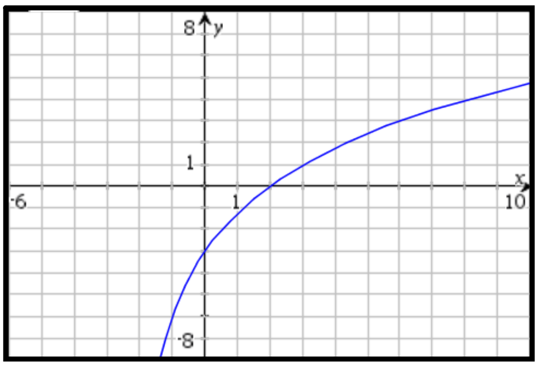
$\log_2 (3x - 4) = 9$

Transformations of logarithmic functions.

Given the graph of $y = \log_2 x$ explain the transformations required to sketch the graph of $y = \log_2(0.5x + 3) + 1$. Compare the graphs with respect to domain, range, intercepts and equation of the asymptotes.



The graph of $y = \log_2 x$ has been transformed into the graph shown below. Write the equation of the transformed graph. (tough one)



Laws of Logarithms.

Evaluate each of the following using the laws of logarithms.

$\log_6 4 + \log_6 9$

$\log_{12} 8 + \log_{12} 9 + \log_{12} 2$

$\log 4000 - \log 4$

$\log_7 100 - \log_7 25 - \log_7 4$

Expand the following.

$$\log \frac{\sqrt{x}\,y^5}{100x}$$

Write as a single logarithm.

$$\log_4 x^2 y^5 + \log_4 xy^{-2} - \log_4 \frac{y^2}{x}$$

Given that $x = \log_5 7$ and $y = \log_5 2$, express the following in terms of x and y.

a. $\log_5 28$

b. $\log_5 70$

Given that $x = \log_4 5$ and $y = \log_4 3$, express $\log_4 225$ in terms of x and y.

If $\log_4 1024 = x + y$ and $\log_8 64 = x - y$, then what is the value of x and y ?

Solving Exponential and Logarithmic Equations.

Solve and check the following equations. Answer to three decimal places if necessary.

a. $125 = 5^x$

b. $4^{2x} = 4096$

c. $9^{x-7} = 27^{2x-9}$

d. $8^{x+2} = \left(\frac{1}{4}\right)^{x+3}$

e. $125 = 25^{\log x}$

f. $3^x = 5$

Solve and check the following equations. Answer to three decimal places if necessary.

a. $\log_2 x = 7$

b. $\log_5 7 = x$

c. $\log_7 x = 2$

d. $\log_x 85 = 3$

e. $\log_5 12 = 2x$

f. $\log_{10} x = 4\log_{10} 2$

Solve and check the following equations. Answer to three decimal places if necessary.

a. $\log_3(4x+9)=5$

b. $\log_2(6x-3)-\log_2 x=4$

c. $\log_8(6x+2)+\log_8(x-3)=2$

d. $3\log_2 x=\log_2 8$

e. $\log_5(x+1)+\log_5(x-3)=1$

f. $\log x-\log 15=\log 0.2$

Solve and check the following equations. Answer to three decimal places if necessary.

a. $\log_4(x+1) - \log_4(2x-3) = \log_4 8$

b. (Challenge) $(\log x)^2 - \log x^3 = 10$

Applications of Exponential and Logarithmic Functions

a. Exponential Growth

$$N = n(G)^{\frac{t}{p}}$$

A type of bacteria doubles every 30 minutes. If there are initially 4 bacteria in the sample, write an exponential function that models the sample's growth over time.

Use your equation to determine the time it takes for the sample to become 4096 bacteria.

A rare coin doubles in value every 12 years. It is currently worth $600. Write a function that models the value of the coin.

Assuming the value continues to grow at a constant rate, use your equation to determine the time needed for the coin to have a value of $2000.

b. Exponential Decay

$$N = n(G)^{\frac{t}{p}}$$

The CANDU (CANadian Deuterium Uranium) reactor is a Canadian-invented pressurized heavy-water reactor that uses uranium-235 (U-235) fuel with a half-life of approximately 700 million years.

What exponential function can be used to represent the radioactive decay of U-235 ?

How long will it take 1 kg of U-235 to decay to 0.125 kg ?

Will the sample above ever decay to 0 kg? Explain.

You have 20 g of phosphorus-32 that decays 5% per day. How long will it take for half the original amount to remain ?

c. Compound Interest

$$A = P(1+i)^n$$

You deposit $1600 in a bank account. Find the balance after 3 years for each of the following situations.

i. The account pays 2.5% annual interest compounded monthly.

ii. The account pays 1.75% annual interest compounded quarterly.

iii. The account pays 4% annual interest compounded semi-annually.

d. Depreciation $V = P(1-r)^t$

You buy a new car for $24000. The value of the car decreases by 16% every year.

i. What will the value of the car be after 2 years ?

ii. When would the car be worth less than $5000 ?

e. <u>Logarithmic Scales</u>

The intensity of sound is measured in decibels (dB). The level of sound, L is given by $L = 10\log\frac{I}{I_0}$, where I is the intensity of the sound and I_0 is the faintest sound detectable to humans.

i. A sound engineer increases the volume at a concert 90 dB to 93 dB. Show that this increase approximately doubles the intensity of the sound.

ii. Determine the level of sound that is 20 times more intense than the threshold of sound, I_0 , correct to the nearest decibel.

iii. The level of sound in a quiet bedroom at night might be 30 dB, while normal conversation has a sound level of about 60 dB. How many times more intense is the normal conversation than the quiet room?

Homework

Unit Test Tomorrow !!

Continue Review:

Text Pages 366 - 367, Exercises # 1 - 12
Text Pages 368 - 369, Exercises # 1 - 14
Text Pages 416 - 418, Exercises # 1 - 23
Text Pages 419 - 420, Exercises # 1 - 17

Attachments

Translations Assignment 1.doc