

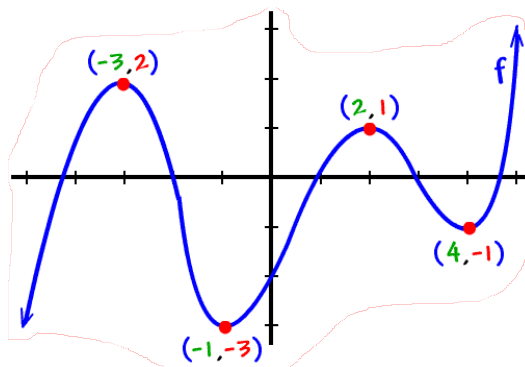
Unit 4: Polynomial and Rational Functions

Lesson 1

$$f(x) = 2x - 1$$

$$f(x) = x^2 - x - 6$$

$$y = x^3 + 2x^2 - 5x - 6$$



Polynomial function - a function of the form:

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_2 x^2 + a_1 x + a_0$$

Polynomial Functions

Polynomial Functions: a function where $f(x)$ is a polynomial expression.

$$y = 2x^4 + 3x - 5$$

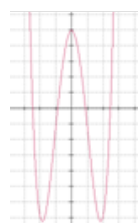
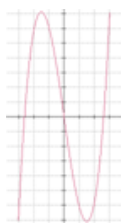
Leading Coefficient: the coefficient of the highest order term in a polynomial

$$y = 2x^4 + 3x - 5$$

Degree of a polynomial: the largest sum of the exponents of the variables in any one term of the polynomial

$$y = 2x^4 + 3x - 5$$

Turning Points: the hills or valleys of a polynomial function are known as its turning points.



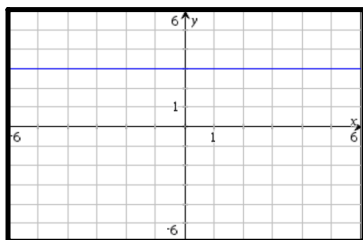
Extends from quadrant: the furthest left quadrant through which the graph travels

Extends to quadrant: the furthest right quadrant through which the graph travels

Characteristics of polynomial functions:

Degree 0: Constant Function

$$f(x) = 3$$

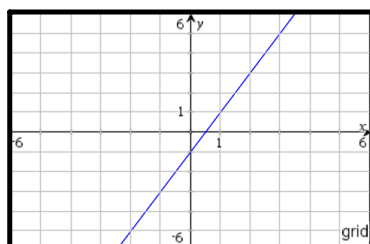


Even Degree
End Behavior

Domain
Range
Number of
x-intercepts

Degree 1: Linear Function

$$f(x) = 2x - 1$$

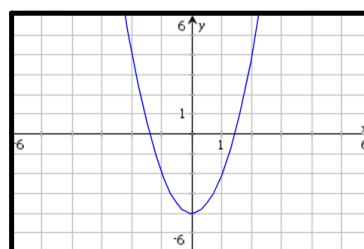


Odd Degree
End Behavior

Domain
Range
Number of
x-intercepts

Degree 2: Quadratic Function

$$f(x) = 2x^2 - 4$$

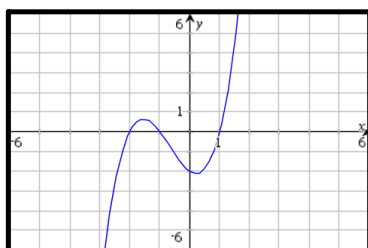


Even Degree
End Behavior

Domain
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Degree 3: Cubic Function

$$f(x) = x^3 + 2x^2 - x - 2$$

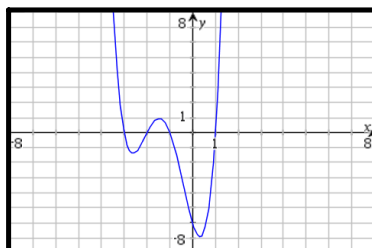


Odd Degree
End Behavior

Domain
Range
Number of
x-intercepts

Degree 4: Quartic Function

$$f(x) = x^4 + 5x^3 + 5x^2 - 5x - 6$$

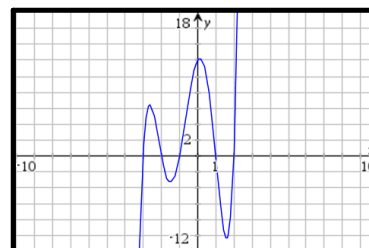


Even Degree
End Behavior

Domain
Range
Number of
x-intercepts

Degree 5: Quintic Function

$$f(x) = x^5 + 3x^4 - 5x^3 - 15x^2 + 4x + 12$$



Odd Degree
End Behavior

Domain
Range
Number of
x-intercepts

Identifying Polynomial Functions:

A Polynomial Function - has whole number exponents
must be able to written as x^n

Example 1: Which functions are polynomials? Justify your answer.

$$g(x) = \sqrt{x} + 5$$

$$f(x) = 3x^4$$

$$y = |x|$$

$$y = 2x^3 + 3x^2 - 4x - 1$$

Try these:

$$h(x) = \frac{1}{x^2}$$

$$f(x) = 3x^2 - 2x^5 + 4$$

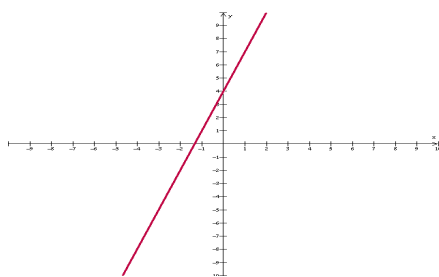
$$y = -4x^4 - 4x + 3$$

$$y = x^{\frac{1}{2}} - 7$$

Example 2: Match a Polynomial with its Graph

For each polynomial, identify the following characteristics:

$$y = 3x + 4$$



Type of function:

Even or odd degree:

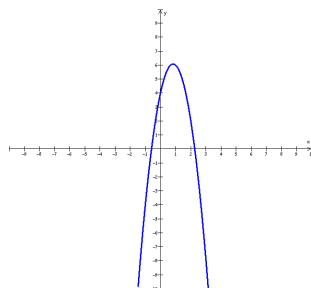
End behaviour:

Number of turning points:

Number of possible x-intercepts:

Does the function have a maximum or minimum value:

$$y = -3x^2 + 5x + 4$$



Type of function:

Even or odd degree:

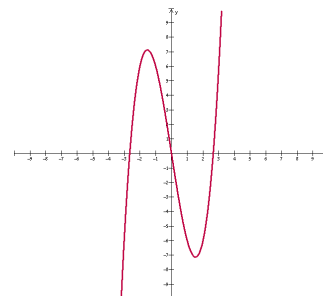
End behaviour:

Number of turning points:

Number of possible x-intercepts:

Does the function have a maximum or minimum value:

$$y = x^3 - 7x$$



Type of function:

Even or odd degree:

End behaviour:

Number of turning points:

Number of possible x-intercepts:

Does the function have a maximum or minimum value:

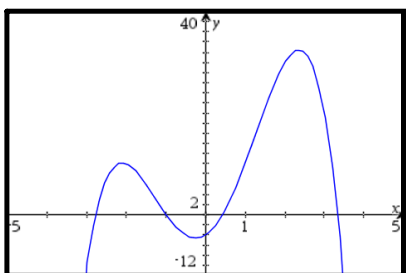
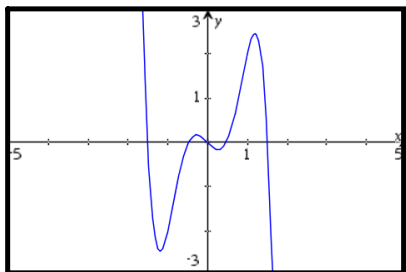
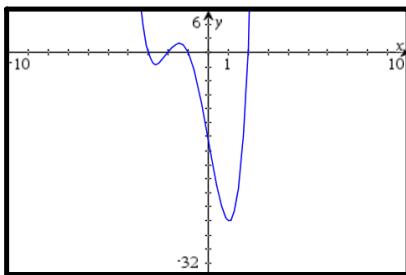
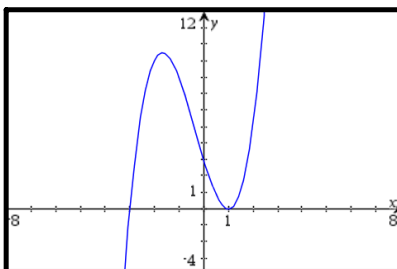
Match the function to its corresponding graph.

$$g(x) = -x^4 + 10x^2 + 5x - 4$$

$$f(x) = x^3 + x^2 - 5x + 3$$

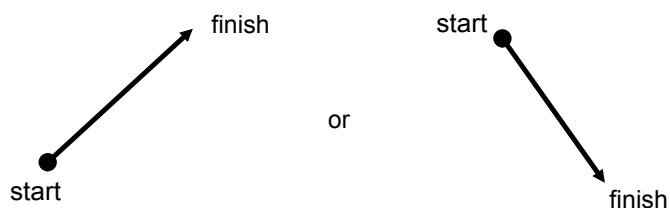
$$p(x) = -2x^5 + 5x^3 - x$$

$$h(x) = x^4 + 4x^3 - x^2 - 16x - 12$$



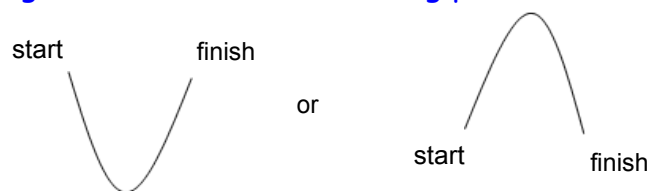
All functions with odd degree will show the following pattern.

$f(x) = x$
 $f(x) = x^3$
 $f(x) = x^5$
ect.



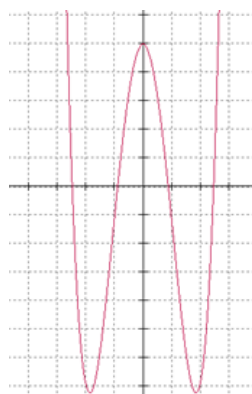
All functions with even degree will follow the following pattern.

$y = x^2$
 $y = x^4$
 $y = x^6$
ect.

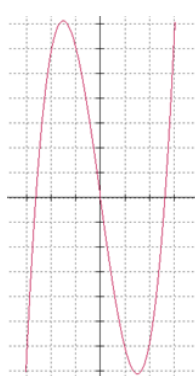


Polynomials functions can have an axis of symmetry, point symmetry or no symmetry.

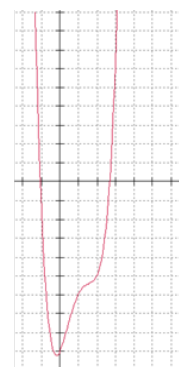
Axis of symmetry



Point Symmetry



No Symmetry



Application of a Polynomial Function:

Example 3: A bank vault is built in the shape of a rectangular prism. Its volume is related to the width, w , in metres, of the vault doorway by the function

$$V(w) = w^3 + 13w^2 + 54w + 72$$

What is the volume, in cubic metres, of the vault if the door is 1 m wide? What is the least volume of the vault? What is the width of the door for this volume? Why is this situation not realistic?

Your Turn: A toaster oven is built in the shape of a rectangular prism. Its volume, V , in cubic inches, is related to the height, h , in inches, of the oven door by the function

$$V(h) = h^3 + 10h^2 + 31h + 30$$

What is the volume, in cubic inches, of the toaster oven if the oven door height is 8 inches? What is the height of the oven door for the least toaster oven volume? Explain.

Homework

1. Assignment Handout;

BLM Polynomial Prerequisite Skills
BLM Polynomial Characteristics

2. Text Pages 114 - 116, Exercises # 1 - 4, 6 - 12, C1, C2a



Attachments

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