The Factor Theorem

Lesson 3

Recall: Remainder for the factor of a polynomial.

Determine the remainder when $x^3 + 2x^2 - 5x - 6$ is divided by $x + 1$

Or

$$x+1\overline{)x^3 + 2x^2 - 5x - 6}$$

$$\begin{array}{r|llll} -1 & 1 & 2 & -5 & -6 \\ - & & & & \\ \hline \times & & & & \end{array}$$

Write the division statement; $\dfrac{P(x)}{x-a} = Q(x) + \dfrac{R}{x-a}$

Factor the quadratic portion of the statement written on the previous slide.

$$x^3 + 2x^2 - 5x - 6 =$$

Write $x^3 + 2x^2 - 5x - 6$ as a product of its three factors.

$$x^3 + 2x^2 - 5x - 6 =$$

What do you notice about the remainder when you divide $x^3 + 2x^2 - 5x - 6$ by any of its three factors?

What does it mean to have a remainder of zero? Graph and find the zeros.

Determining Factors.

Which of the following are factors of $P(x) = x^3 - 7x + 6$

Use the Remainder Theorem:

$x + 1$

$x - 1$

$x + 2$

$x - 2$

$x + 3$

$x - 3$

The Factor Theorem states that a polynomial in x, P(x), has factor (x - a) if and only if P(a) = 0

Example 1:

Use the Factor Theorem to Test for Factors of a Polynomial

Which binomials are factors of the polynomial $P(x) = x^3 - 3x^2 - x + 3$
Justify your answers.

Potential factors are $\pm 1, \pm 3$ (these are the factors of the constant)

$x - 1$

$x + 1$

$x - 3$

$x + 3$

Your Turn:

Which binomials are factors of the polynomial $P(x) = x^3 + 2x^2 - 5x - 6$
Justify your answers.

Potential factors are
(these are the factors of the constant)

$x - 1$

$x + 1$

$x - 2$

$x + 2$

$x - 3$

$x + 3$

$x - 6$

$x + 6$

Example 2: Factor Using the Integral Zero Theorem

Integral Zero Theorem:

If x = a is an integral zero of a polynomial, P(x), with integral coefficients, then a is a factor of the constant term of P(x).

This theorem helps us to identify which integer values of a to try when determining if P(a) = 0

Factor $2x^3 - 5x^2 - 4x + 3$ fully.

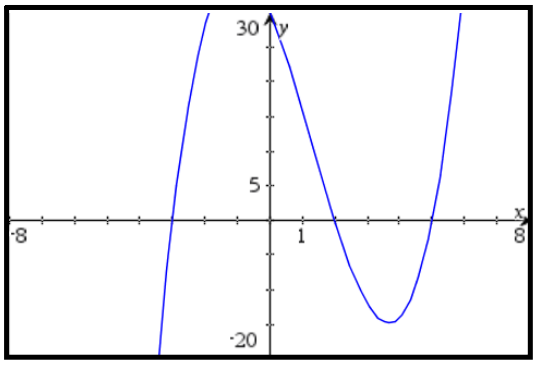
How can we use the factors of the polynomial expression to determine the zeros of the corresponding polynomial function.

Graph the function and test your hypothesis. $y_1 = 2x^3 - 5x^2 - 4x + 3$

Your Turn:

What is the fully factored form of $x^3 - 4x^2 - 11x + 30$?

How could you simplify your search using the graph?



Factor Higher Degree Polynomials:

Example 3: Fully factor $x^4 - 5x^3 + 2x^2 + 20x - 24$

After finding one factor, you can,

Use division to find the other factors.

Or apply the factor theorem again.

Or factor by grouping.

Your Turn:

Fully factor $x^4 - 3x^3 - 7x^2 + 15x + 18$

Example 4: Solve Problems Involving Polynomial Expressions

A shipping container that has the shape of a rectangular prism, has a volume, in cubic feet, represented by the polynomial function,

$$V(x) = x^3 + 7x^2 - 28x + 20$$

where x is a positive real number.

What are the factors that represent possible dimensions, in terms of x , of the container?

Method I: Use factoring,

Method II: Use graphing.

Example 5:

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The product of four integers is $x^4 + 6x^3 + 11x^2 + 6x$, where x is one of the integers. What are possible expressions for the other three integers?

Homework

1. Assignment Handout; BLM: The Factor Theorem

2. Text Pages 133 - 135, Exercises # 1 - 3 ace, 4, 5 - 6 ace, 7 - 11, C3

Attachments

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