Equations and Graphs of Polynomial Functions

Lesson 4

Recall: the zeros of any polynomial function, $y = f(x)$, are the roots of the corresponding equation, $f(x) = 0$, and the x-intercepts of the graph.

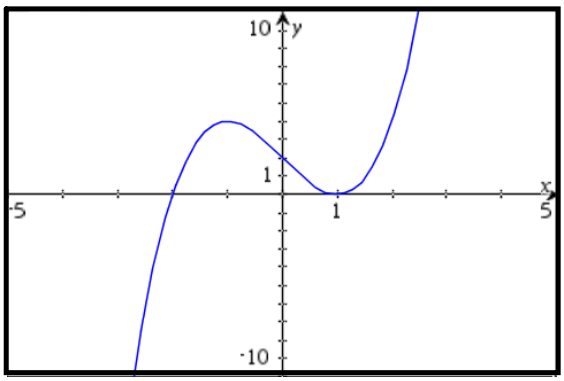
For example, the function $f(x) = (x-1)(x-1)(x+2)$ has two identical zeros at $x = 1$ and a third zero at $x = -2$. These are the roots of the equation $(x-1)(x-1)(x+2) = 0$

If a polynomial has a factor $(x-a)$ that is repeated n times, then $x = a$ is a zero of multiplicity, n .

The function, $f(x) = (x-1)^2(x+2)$ has a zero of multiplicity 2 at $x = 1$

The equation $(x-1)^2(x+2) = 0$ has a root of multiplicity 2 at $x = 1$

Consider the graph of the above function $f(x) = (x-1)(x-1)(x+2)$



Notice that the sign of the function at the zero $x = -2$ (zero of odd multiplicity), changes as the graph crosses the x-axis.

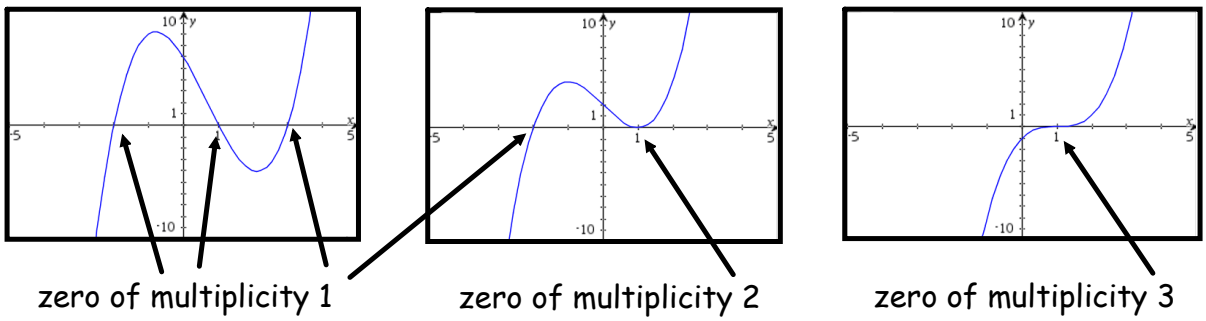
The sign of the function doesn't change at $x = 1$ (zero of even multiplicity) and the graph 'bounces' off the x-axis.

Multiplicity (of a zero)

The number of times a zero of a polynomial occurs.

The shape of the graph of a function close to its zero depends on its multiplicity.

The multiplicity of a zero or root can also be referred to as the order of the zero or root.



Example 1: Analysing the Graphs of Polynomial Functions.

For each graph of a polynomial function, determine,

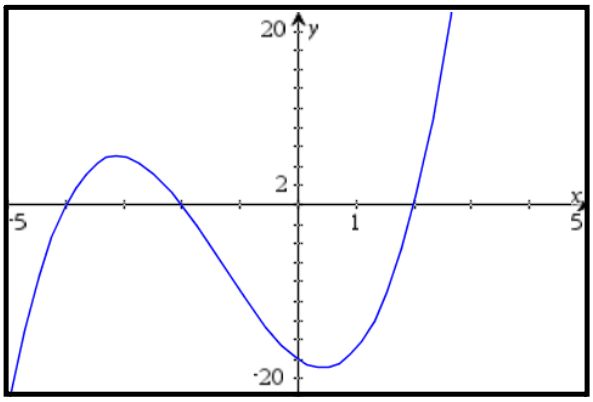
the least possible degree

the sign of the leading coefficient

the x-intercepts

the factors of the function

the intervals where the function is positive or negative



the least possible degree

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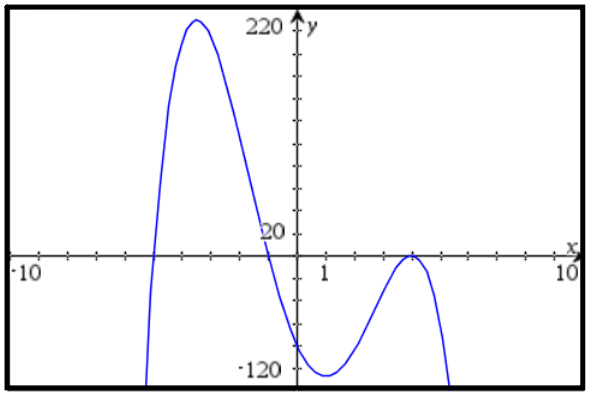
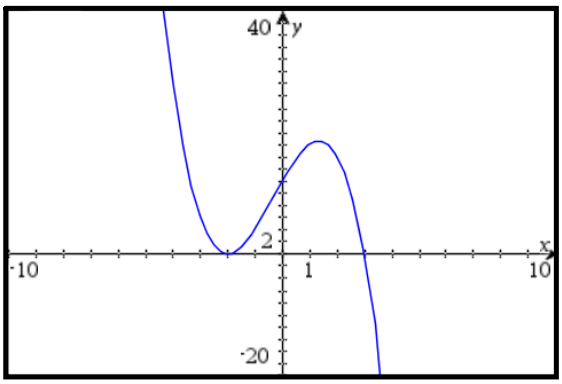


Your Turn:

For the graph of a polynomial function, determine,



the least possible degree

the sign of the leading coefficient

the x-intercepts

the factors of the function

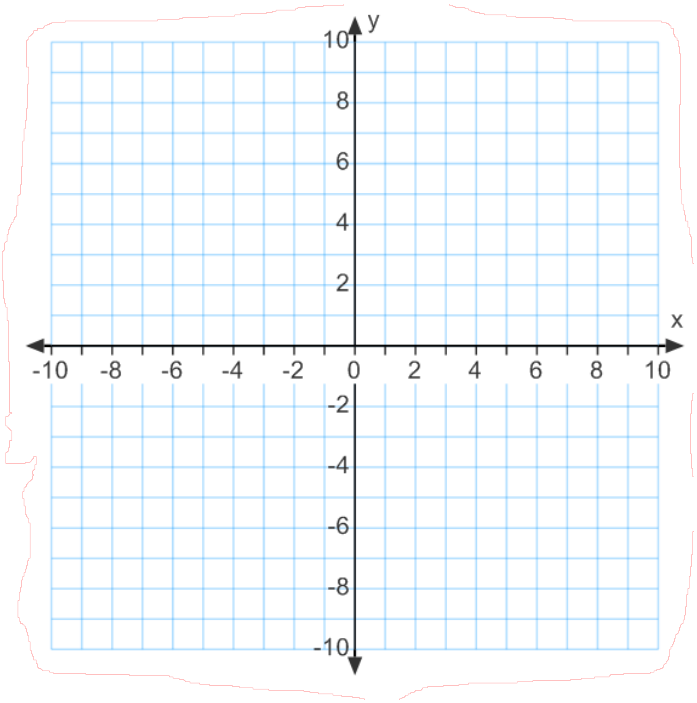
the intervals where the function is positive or negative

Example 2: Analyse Equations to Sketch the Graphs of Polynomial Functions

Sketch the graph of the polynomial function $y = -(x+1)^3(x-3)$

The function $y = -(x+1)^3(x-3)$ is already_____.

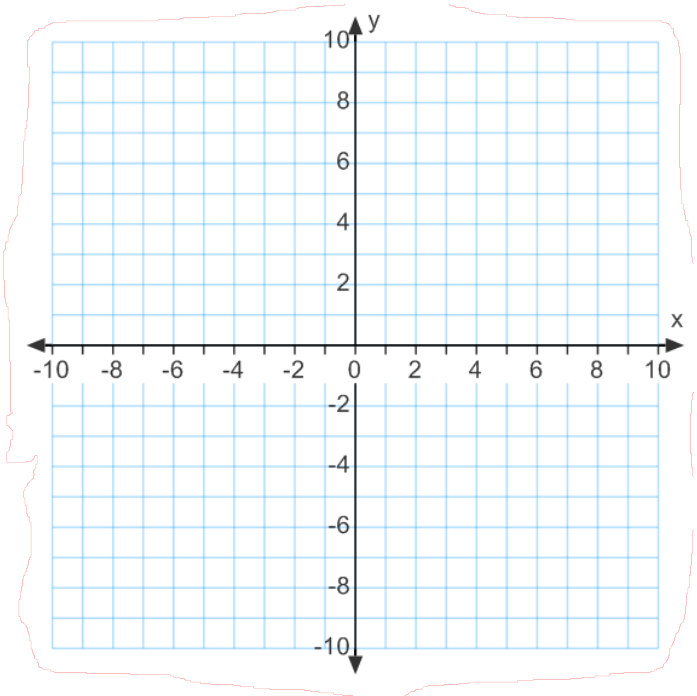
Degree	
Leading coefficient	
End behavior	
Zeros/x-intercepts	
Multiplicity of zeros	
y-intercept	
Intervals where the function is positive.	
Intervals where the function is negative	



Sketch the graph of the polynomial function $y = 2x^5 + x^4 - 18x^3 - 9x^2$

The function $y = 2x^5 + x^4 - 18x^3 - 9x^2$ needs to be factored.

Degree	
Leading coefficient	
End behavior	
Zeros/x-intercepts	
Multiplicity of zeros	
y-intercept	
Intervals where the function is positive.	
Intervals where the function is negative	



Applying Transformations to Sketch a Graph:

Example 3: The graph of $y = x^4$ is transformed to obtain the graph of

$$y = 3\left[\frac{1}{2}(x+1)\right]^4 - 4$$

State the parameters and describe the corresponding transformations

a =

b =

h =

k =

Use mapping notation to show what happens to points under transformation.

$(x, y) \rightarrow (\quad , \quad)$

$y = x^4$ $\qquad y = 3\left[\frac{1}{2}(x+1)\right]^4 - 4$

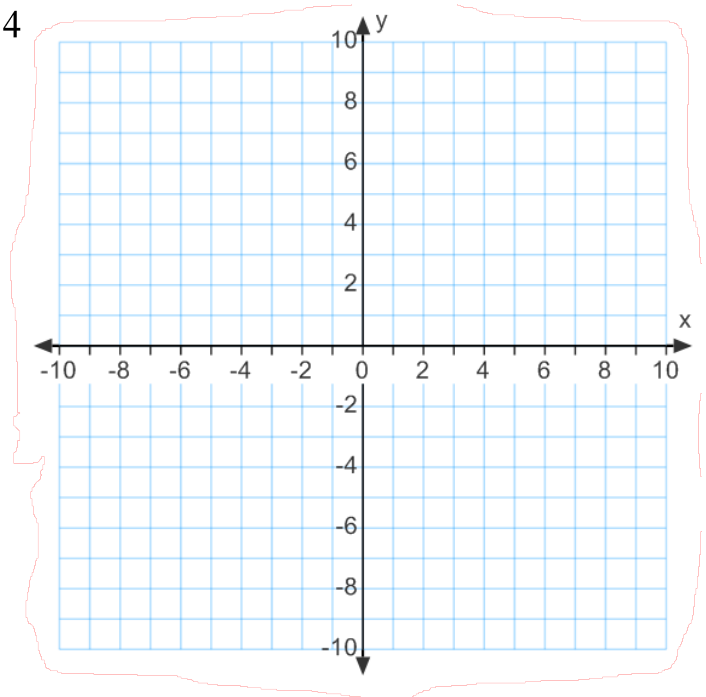
$(-2, 16) \rightarrow (\quad , \quad)$

$(-1, 1) \rightarrow (\quad , \quad)$

$(0, 0) \rightarrow (\quad , \quad)$

$(1, 1) \rightarrow (\quad , \quad)$

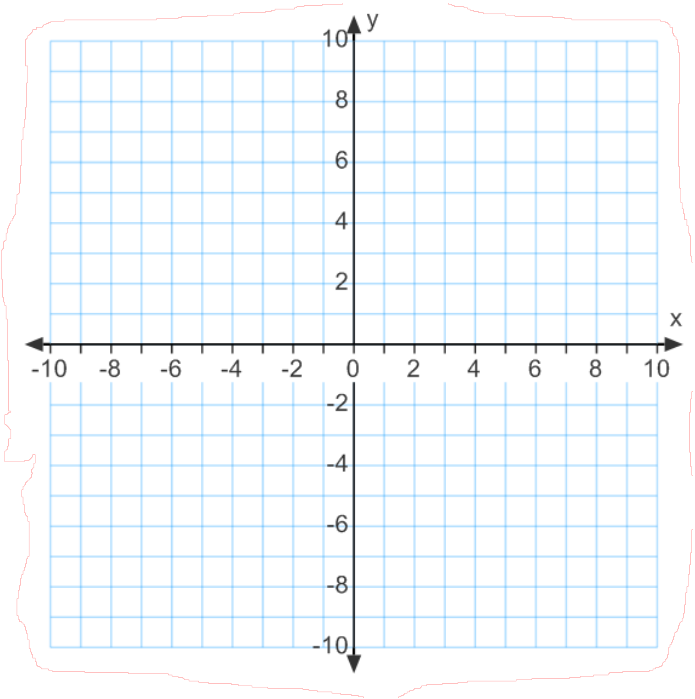
$(2, 16) \rightarrow (\quad , \quad)$



Remember, SRT!

Your Turn:

Transform the graph of $y = x^3$ to sketch the graph of $y = -4\left[2\left(x+2\right)\right]^3 - 5$



Example 4: Solve Problems Involving Polynomial Functions

Three consecutive integers have a product of -210 .

Write a polynomial function to model this situation.

What are the three integers?

Example 5: Determine the equation of each polynomial function.

a. Quartic function with zeros 2 (multiplicity 3), and -5, and a y-intercept of 30.

b. Quintic function with zeros -1 (multiplicity 2), 3 (multiplicity 1) and -2 (multiplicity 2), and a constant term -12.

Homework

1. Assignment Handout; BLM: Graphing Polynomials

2. Text Pages 147 - 152, Exercises # 1 - 7, 9, 10, 12 - 16, 22, 23, C2, C3

Attachments

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