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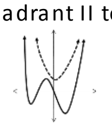
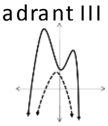
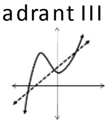
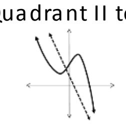
Study Smarter, Do Better

Function Type		Original Function	Horizontal Stretch	Vertical Stretch	Horizontal Translation	Vertical Translation	Combined Transformation
General Function	Form	$y=f(x)$	$y=f(bx)$	$y=af(x)$	$y=f(x-h)$	$y=f(x)+k$	$y=af\left(b(x-h)\right)+k$
General Function	Effect	No Effect	Horizontal Stretch By Factor $\frac{1}{\lvert b\rvert}$ Wider: $0<\lvert b\rvert<1$ Narrower: $\lvert b\rvert>1$ Reflect $y-axis$: $b<0$	Vertical Stretch By Factor $\lvert a\rvert$ Shorter: $0<\lvert a\rvert<1$ Taller: $\lvert a\rvert>1$ Reflect $x-axis$: $a<0$	$h>0$ Shift Right $h<0$ Shift Left	$k>0$ Shift Up $k<0$ Shift Down	Apply Transformations in BEDMAS Order (Stretches and reflections, then translations)
General Function	Mapped To	$(x,y)\to(x,y)$	$(x,y)\to\left(\frac{1}{b}x,y\right)$	$(x,y)\to(x,ay)$	$(x,y)\to(x+h,y)$	$(x,y)\to(x,y+k)$	$(x,y)\to\left(\frac{1}{b}x+h,ay+k\right)$
Radical		$y=\sqrt{x}$	$y=\sqrt{bx}$	$y=a\sqrt{x}$	$y=\sqrt{x-h}$	$y=\sqrt{x}+k$	$y=a\sqrt{b(x-h)}+k$
Sinusiodal		$y=\sin\theta$ $y=\cos\theta$	$y=\sin b\theta$ $y=\cos b\theta$	$y=a\sin\theta$ $y=a\cos\theta$	$y=\sin(\theta-h)$ $y=\cos(\theta-h)$	$y=\sin\theta+k$ $y=\cos\theta+k$	$y=a\sin\left(b(\theta-h)\right)+k$ $y=a\cos\left(b(\theta-h)\right)+k$
Exponential		$y=c^x$	$y=c^{bx}$	$y=ac^x$	$y=c^{x-h}$	$y=c^x+k$	$y=ac^{b(x-h)}+k$
Logarithmic		$y=\log x$	$y=\log bx$	$y=a\log x$	$y=\log(x-h)$	$y=\log x+k$	$y=a\log\left(b(x-h)\right)+k$

Polynomial Functions

Form: $f(x)=a_nx^n+a_{n-1}x^{n-1}+a_{n-2}x^{n-2}+\cdots+a_2x^2+a_1x^1+a_0$

Even Degree		Odd Degree	
0 to maximum of n $x-$intercepts Domain $\{x\mid x\in R\}$ Range depends on max and min y-intercept a_o Maximum and Minimum		At least 1 to maximum of n $x-$intercepts Domain $\{x\mid x\in R\}$ Range $\{y\mid y\in R\}$ y-intercept a_o No max or min	
$a_n>0$	$a_n<0$	$a_n>0$	$a_n<0$
Quadrant II to I 	Quadrant III to IV 	Quadrant III to I 	Quadrant II to IV 

Radical Functions

Domain of $y=\sqrt{f(x)}$ any value $f(x)\geq 0$

Range of $y=\sqrt{f(x)}$ any value $\sqrt{f(x)}$ is defined

$f(x)<0$	$f(x)=0$	$0<f(x)<1$	$f(x)=1$	$f(x)>1$
$y=\sqrt{f(x)}$ Undefined	$y=\sqrt{f(x)}$ and $y=f(x)$ intersect at $x=0$	$y=\sqrt{f(x)}$ above $y=f(x)$	$y=\sqrt{f(x)}$ and $y=f(x)$ intersect at $f(x)=1$	$y=\sqrt{f(x)}$ below $y=f(x)$

Solving Radical Functions Algebraically

1) List Restrictions
2) Isolate Radical and Square Both Sides
3) Find Roots of Equation (Solve for x)
4) Check solutions in original equation

Inverse of Function

Denoted by: $y=f^{-1}(x)$

Swap x and y $\begin{cases}(x,y)\to(y,x)\\ y=f(x)\to x=f(y)\end{cases}$

Factor Theorem

- If x - a is a factor then $P(a)=0$
- If $P(a)$ then x - a is a factor of $P(x)$

Remainder Theorem

$$\frac{P(x)}{x-a}=Q(x)+\frac{R}{x-a}$$

$x+1$ into x^3+3x^2-2x+5

$$\begin{array}{r|rrrr} 1 & 1 & 3 & -2 & 5 \\ & \downarrow & 1 & 2 & -4 \\ \hline & 1 & 2 & -4 & 9 \end{array}$$

$$\frac{x^3+3x^2-2x+5}{x+1}$$

$$=x^2+2x-4+\frac{9}{x+1}$$

Multiplicity of Zero

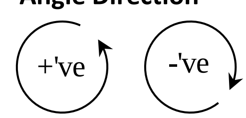
Polynomial has factor x - a repeated n times, $x=a$ is zero of multiplicity n

ex)

$y=(x-2)^2$

$x=2$ is a zero multiplicty of 2

Angle Direction



Logarithmic and Exponential Modeling

Model Real Life Problems Using

Final Quantity=initial quantity $\times$(change factor)$^{\text{number of changes}}$

$y=$ Final Quantity

$a=$ initial quantity

$b=$ change factor

$x=$ number of changes

$$y=a\bullet b^x$$

Sine Law

$$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$$

Cosine Law

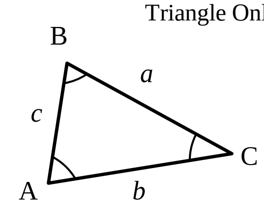
$$a^2=b^2+c^2-2ab\cos A$$

NOTE: Sine and Cosine Law True For Any Triangle

Pythagorean Theorem

$$a^2=b^2+c^2$$

Right Angle Triangle Only



Trig Identities

SOH CAH TOA

$$\sin\theta = \frac{opposite}{hypotenuse}$$

$$\cos\theta = \frac{adjacent}{hypotenuse}$$

$$\tan\theta = \frac{opposite}{adjacent}$$

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

Sum

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1-\tan A\tan B}$$

Difference

$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$

$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A\tan B}$$

Exponent Rules

$$a^b a^c = a^{b+c} \qquad (a^b)^c\ a^{bc} \qquad \frac{a^b}{a^c} = a^{b-c}$$

$$a^{-b} = \frac{1}{a^b}$$

Double Angle

$$\sin 2A = 2\sin A\cos B$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\cos 2A = 2\cos^2 A - 1$$

$$\cos 2A = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

Log Rules

$$\log_c MN = \log_c M + \log_c N$$

$$\log_c \frac{M}{N} = \log_c M - \log_c N$$

$$\log_c M = \frac{\log_a M}{\log_a C}$$

Function Notation

$$h(x) = f(x) + g(x)$$
$$h(x) = (f+g)(x)$$

$$h(x) = f(x) - g(x)$$
$$h(x) = (f-g)(x)$$

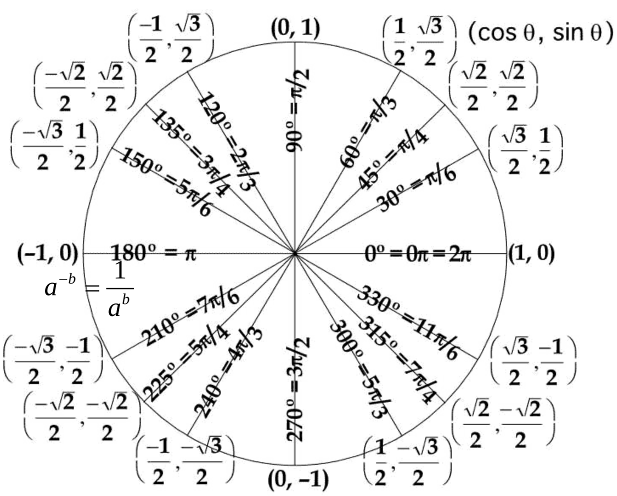
$$h(x) = f(x)\bullet g(x) = (f\bullet g)(x)$$

$$h(x) = \frac{f(x)}{g(x)}$$

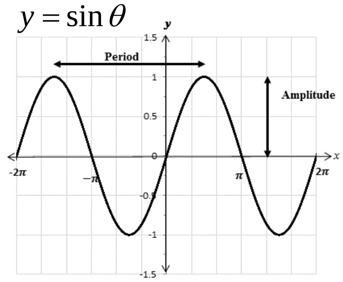
$$h(x) = \left(\frac{f}{g}\right)(x) \quad g(x) \neq 0$$

$$h(x) = f(g(x)) = (f\circ g)(x)$$

Trigonometry and Unit Circle



$y = \sin\theta$



Max is +1 and Min is -1

Amplitude is 1

Period is 2π $\qquad$ $\text{Period} = \frac{2\pi}{b}$ or $\frac{360^\circ}{b}$

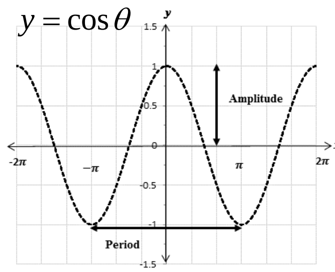
y-intercept is 0

θ intercepts at $-\pi, 0, \pi, 2\pi$

Domain $\{\theta \mid \theta \in R\}$

Range $\{y \mid -1 \le y \le 1, y \in R\}$

$y = \cos\theta$



Max is +1 and Min is -1

Amplitude is 1

Period is 2π $\qquad$ $\text{Amplitude} = \frac{\text{Max Value - Min Value}}{2}$

y-intercept is 1

θ intercepts at $-\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$

Domain $\{\theta \mid \theta \in R\}$

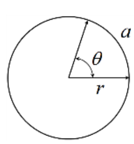
Range $\{y \mid -1 \le y \le 1, y \in R\}$

Coterminal Angle

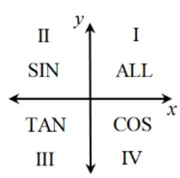
$$\theta \pm 360^\circ n, n \in N$$

$$\theta \pm 2\pi n, n \in N$$

$$a = r\theta$$



CAST rule



Permutations and Combinations

$$n! = n\times(n-1)\times(n-2)\times\cdots\times3\times2\times1$$

n objects with

a of one kind that are identical and

b of second kind that are identical

can be arranged in $\frac{n!}{a!b!}$ ways

NOTE

$0! = 1$

$${}_nP_r = \frac{n!}{(n-r)!}, n \in N$$

n objects to choose from

r number of items taken

Order MATTERS

$${}_nC_r = \frac{n!}{(n-r)!r!}, n \in N$$

n objects to choose from

r number of items taken

Order DOESN'T matter

Binomial Theorem

Coefficients of terms = $(n+1)$ row of Pascal's Triangle

Binomial	Pascal's Triangle in Binomial Expansion	Row
$(x+y)^0$	1	1
$(x+y)^1$	1x + 1y	2
$(x+y)^2$	$1x^2 + 2xy + 1y^2$	3
$(x+y)^3$	$1x^3 + 3x^2y + 3xy^2 + 1y^3$	4
$(x+y)^4$	$1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$	5

Pascal's Triangle	Combinations
1	${}_0C_0$
1 1	${}_1C_0$ ${}_1C_1$
1 2 1	${}_2C_0$ ${}_2C_1$ ${}_2C_2$
1 3 3 1	${}_3C_0$ ${}_3C_1$ ${}_3C_2$ ${}_3C_3$
1 4 6 4 1	${}_4C_0$ ${}_4C_1$ ${}_4C_2$ ${}_4C_3$ ${}_4C_4$
1 5 10 10 5 1	${}_5C_0$ ${}_5C_1$ ${}_5C_2$ ${}_5C_3$ ${}_5C_4$ ${}_5C_5$

Properties of Binomial Expansion of $(x+y)^n$ $n \in \mathrm{N}$

Write Expansion in descending order of exponent of first term (In this case x)

Term number is $k+1$

Expansion contains $n+1$ terms

Sum of exponents in any term n

General Term Number

$$t_{k+1} = {}_nC_k(x)^{n-k}(y)^k$$

 9/12/13