MATH 30-2 Short Answer and Problems (taken from Alberta Educatio Assessment Standards and Exemplars):

Use the following information to answer the next question.

The goal of a particular puzzle is to fill the circles in a grid with the letters $\mathrm{A}, \mathrm{B}, \mathrm{C}$, and D so that no letters are repeated in any row, column, or set of connected circles.

The three entries in the grey circles were given to start the puzzle. Jerome has already completed three entries shown in the white circles, but he has made an error.

1)(SE) Identify the error that Jerome made in his solution to the puzzle:
b) Explain why this entry is incorrect:
c) Correct the error that Jerome made and finish the puzzle:

Use the following information to answer the next four questions.
A group of 100 students was surveyed about movies that they have seen, as shown below.
2 people saw all three movies
12 people saw "Metal Man" and "The Princely Groom"
53 people saw "Metal Man"
10 people saw "Metal Man" and "Quick and Angry 8"
18 people saw "The Princely Groom" only
23 people saw "The Princely Groom" and "Quick and Angry 8"
6 people did not see any of the movies
Jason started to organize the results in the Venn diagram shown below.

2)(SE) Fill in the Venn Diagram.
3)

A class of 35 students has 17 males. One student will be selected at random from the class. Jeanette suggested that the odds in favour of selecting a male student would be 17:35. Is Jeannette correct? Justify your answer.

Use the following information to answer the next question.
A television game show has listed the following odds in favour of winning for three of their games.

| Game | Odds of Winning |
| :--- | :---: |
| Flip'em | $1: 3$ |
| Central Eye | $2: 5$ |
| Minefield | $1: 4$ |

4. a. What is the probability of winning the Flip 'em game?
b. Which of the 3 games is a contestant most likely to win? Justify your answer.
5. 

At a private school, each student must wear a school uniform that includes a dress shirt and pants. The dress shirt can be white, gray, or light blue. The pants can be navy or black. Use a graphic organizer to show the different possible variations of the uniform.

Use the following information to answer the next question.
For the set of numbers 1 to 20 inclusive, Theresa knows that some numbers are divisible by 3 and some numbers are even. She is going to write each number on a ball and place them in a box.
6.(SE) a. Model this set using a graphic organizer (possibly Venn Diagram. . .)
b. If 1 ball is randomly selected from the box, what is the probability that the number written on it is divisible by 3 OR is an even number?
c. Explain why 14 and 17 would be examples of numbers that belong to the set $P($ Divisible by $3 \cap$ even number)?

Use the following information to answer the next question.

The probability of Brenda getting a hit in a baseball game is 0.345 . The probability of Brenda or Deborah getting a hit during the game is 0.617 . The probability of both Brenda and Deborah getting hits during the game is 0.224 .
12. Determine the probability of Deborah getting a hit in the game:
13. Determine the \# of arrangements of all the letters in the word TATTOO.
14. Determine the \# of 3 letter arrangements of the letters of the word DIPLOMA.
15.

Only six people have tickets for 2 prizes in a school draw. Once a ticket is drawn for a prize, it is not reentered in the draw. What is the probability that Bill wins the first prize and Mary wins the second prize?
16. A 7-player volleyball team must start in a straight line for a picture.
a) Determine the \# of different arrangements that can be made for the picture.
b) Determine the \# of different arrangements that can be made for the picture if the tallest player must stand in the middle.
17. In a group of 9 students, there are 4 females and 5 males.
a) (SE) How many different committees consist of 3 OR 4 students?
b) How many different 4 member committees have 2 females and 2 males?
c) (SE) Determine the probability that a 4-member committee chosen at random from this group will consist of 2 males and 2 females:
18.

Ralph knows that there are 15 distinguishable possibilities when 2 people are chosen to form a committee from a particular group of $n$ people.
a. Describe what values of $n$ would be admissible in this problem.
b. Determine the \# of people in the larger group, n.

Use the following information to answer the next question.

Sanja and David both simplified the expression $\frac{x}{x^{2}+x}$. Their work is shown below.

| Sanja | David |
| :--- | :--- |
| $\frac{x}{x^{2}+x}$ | $\frac{x}{x^{2}+x}$ |
| $\frac{x}{x(x+1)}$ | $\frac{1 x}{x^{z}+1 x}$ |
| $\frac{1}{x+1}$ | $\frac{1}{x+1}$ |

Sanja stated that the non-permissible values of $x$ for the equivalent rational expressions are -1 and 0 .

David stated that the non-permissible value of $x$ for the equivalent rational expressions is -1 .
19. Which student is correct? Justify your choice.
20.

Explain why the non-permissible value for the expression $\frac{3 x}{(x+2)}$ is -2 .
21.

Simplify the following. State all non-permissible values.
a. $\frac{5}{3 x^{2}} \cdot \frac{6 x}{x+2}$
b. $\frac{x}{x+2} \cdot \frac{x+2}{x-3}$
c. $\frac{x+3}{5 x-1} \div \frac{2 x+6}{4 x}$
d. $\frac{x^{2}+3 x}{x^{2}-4} \div \frac{x+3}{x+2}$
22.

Simplify the following. State all non-permissible values.
a. $\frac{3}{5 x}+\frac{7 x}{4}$
b. $\frac{4 x}{x+2}-\frac{5 x+3}{x+2}$
c. $\frac{x}{3-x}-\frac{3}{x-3}$
d. $\frac{x}{x^{2}-4}+\frac{3 x}{x^{2}+2 x}$
e. $\frac{x^{2}+3 x}{x^{2}-4}+\frac{x^{2}+5 x}{x+2}$
23.

Solve each equation.
a. $\frac{5 x-1}{4 x+11}=\frac{3}{4}$
b. $\frac{3}{x}+\frac{5}{3}=10$
c. $\frac{4}{x}+\frac{6 x}{x+1}=6$
d. $\frac{2 x}{x+3}+\frac{x}{x-3}=\frac{18}{x^{2}-9}$
24. (SE)

Use the following information to answer the next question.
A student solved a rational equation using the steps shown below.

$$
\frac{x}{x+1}-\frac{3}{x-2}=-9, x \neq-1,2
$$

Step $1 \quad x(x-2)-3(x+1)=-9$

Step $2 x^{2}-2 x-3 x-3=-9$

Step $3 x^{2}-5 x+6=0$
Step $4(x-3)(x-2)=0$
Step $5 \quad x=3,2$

Identify the errors made in the steps shown above, and justify the corrections necessary to obtain the correct solution.
25. Write $4^{2}=16$ in logarithmic form.
26.

Evaluate $\log _{2}\left(\frac{1}{16}\right)$.

Write each of the following logarithmic equations in exponential form.
a. $\quad \log (100)=2$
b. $\quad \log _{2} 8=3$
c. $\ln (x)=2$
d. $\log _{a} 5=2$
28.

Use the laws of logarithms to determine the value of each of the following.
a. $\quad \log _{6} 3+\log _{6} 12$
b. $\log 520-\log 52$
29. Describe how to estimate the value of $\log _{2} 15$ without using technology.
30. (SE) Express $2 \ln x-\ln y$ as a single logarithm.
31. Express $\log 6$ in a different logarithmic form.
32.

## Solve algebraically.

a. $3=9^{2 \mathrm{x}}$
b. $\quad 2^{(x-1)}=4^{(x-2)}$
c. $\quad 10=3^{\mathrm{x}}$
d. $\quad 2^{(x-1)}=3$
(SE)
33. Describe how to determine the solution $2^{(x-1)}=3^{(x-2)}$ graphically.

Use the following information to answer the next question.
Sam deposits $\$ 100$ into a savings account that pays $2.4 \% / \mathrm{a}$, compounded monthly. A function that models the growth of the deposit is

$$
y=100\left(1+\frac{0.024}{12}\right)^{x}
$$

where $x=$ number of months and $y=$ value of investment, in dollars.
34. a) Determine how long it will take for the investment to be worth atleast $\$ 150$ at 2.4\%/a, compounded monthly.
b) (SE) Modify the exponential function to reflect an interest rate of $4 \% / \mathrm{a}$, compounded quarterly.

Use the following information to answer the next question.
A researcher discovered mould growing in a Petri dish in her laboratory. When first observed, the mould covered only $3 \%$ of the dish's surface. Every 24 hours, the surface area of the mould doubles in size, as shown in the table below.

| Time (h) | Area covered (\%) |
| :---: | :---: |
| 0 | 3 |
| 24 | 6 |
| 48 |  |
| 72 |  |

35. a) Complete the table above and then write an exponential function to model the growth of the mould over time.
b) Use your function from part a) to determine the approximate length of time, to the nearest tenth of an hour, it will take for the petri dish to be completely covered with mould.
36. 

Use the following information to answer the next question.
Corlene invested money in a GIC that pays interest compounded annually, as shown in the table below.

| Year | Value of Investment |
| :---: | :---: |
| 0 | $\$ 1000.00$ |
| 1 | $\$ 1020.00$ |
| 2 | $\$ 1040.40$ |
| 3 | $\$ 1061.21$ |

a) To model the investment's growth and predict it's future value, Corlene has chosen to use an exponential model. Is this an effective model?
b) Write an exponential function that Corlene could use to predict the future value of her investment. Explain what the numerical values in your function represent in the context of this problem.
c) (SE) If Corlene invested in a GIC that paid 1.40\%/a compounded annually, how would this affect the value of the investment over time?
37.

Use the following information to answer the next question.

When objects of different mass are compared without a scale, to be perceived the difference in mass must be large enough. For example, when held in a person's hands, masses within 5 g of 100 g will seem to be the same. The 5 g difference is known as the Minimum Perceivable Difference.

For heavier objects, the Minimum Perceivable Difference increases. The Minimum Perceivable Difference for various masses is shown in the table below.

| Mass (g) | Minimum Perceivable <br> Difference $(\mathbf{g})$ |
| :---: | :---: |
| 100 | 5 |
| 200 | 10 |
| 400 | 15 |
| 800 | 20 |

These data can be modelled by a logarithmic regression function of the form

$$
y=a+b \ln (x)
$$

where $x$ is the mass of the object, in grams, and $y$ is the Minimum Perceivable Difference in mass, in grams.
a) Determine the logarithmic regression function of the form $y=a+b \ln (x)$, to model these data. Round values of $a$ and $b$ to the nearest tenth.
b) Based on the regression equation, determine the Minimum Perceivable Difference for a 2100 g object, to the nearest whole gram.

Use the following information to answer the next question.
A hockey arena seats 1600 people. The cost of a ticket is $\$ 10$. At this price, every ticket is sold. To obtain more revenue, the arena management plans to increase the ticket price. A survey was conducted to estimate the potential revenue for different ticket prices, as shown below.

| Ticket price (\$) | Potential Revenue (\$) |
| :---: | :---: |
| 10 | 16000 |
| 15 | 19500 |
| 20 | 20300 |
| 25 | 14750 |
| 30 | 5500 |

The data above can be modelled by a quadratic regression function of the form

$$
y=a x^{2}+b x+c
$$

where $x$ is the ticket price, in dollars, and $y$ is the potential revenue, in dollars.
38. Determine the ticket price that would maximize the revenue.

Use the following information to answer the next question.
A Ferris wheel has a radius of 8 m and its centre is 10 m above the ground. A rider gets on a chair of the Ferris wheel at its lowest point and completes one full revolution in 48 seconds.
39. a) (SE)

Sketch a graph of the rider's height above the ground over the first 48 seconds on the grid below and label key points on the graph.

b) State the amplitude,period, and equation of the midline for the function sketched in part a) above.
c) Determine a function of the form $h=a \sin (b t-1.57)+d$, where $h$ represents the height of a rider above the ground and and $t$ represents the time after the ride has started that could be used to model the height of a rider on the Ferris wheel described above.
40.

Use the following information to answer the next question.
The average daily high temperature of Montreal, in ${ }^{\circ} \mathrm{F}$, for each of the months of the year is shown in the table below. January is month 1, February is month 2, etc..

| Month | Average Daily High <br> Temperature in ${ }^{\circ} \mathbf{F}$ | Month | Average Daily High <br> Temperature in ${ }^{\circ} \mathbf{F}$ |
| :---: | :---: | :---: | :---: |
| 1 | 22 | 7 | 80 |
| 2 | 25 | 8 | 77 |
| 3 | 36 | 9 | 67 |
| 4 | 52 | 10 | 51 |
| 5 | 66 | 11 | 41 |
| 6 | 75 | 12 | 28 |

Write a sinusoidal regression function of the form $y=a \cdot \sin (b x+c)+d$, where $x$ is the month number and $y$ is the average daily high temperature, that could be used to model these data. Round the values of $a, b, c$, and $d$ to the nearest hundredth.

Use the following information to answer the next question.
The graph of a sinusoidal function is shown below.

41.

The amplitude of the sinusoidal function is $\qquad$ $i$ units and the midline is $y=$ $\qquad$ units.

Pick one of the following choices to fill in each blank in the statement above:
Choices for amplitude in blank $i$ are: 10, 20, 30, 40, or 50
Choices for midline in blank $i \boldsymbol{i}$ are: $10,20,30,40$, or 50

## Use the following information to answer the next question.

The graph of a sinusoidal function is shown below. The points $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}$, and $\boldsymbol{E}$ are labelled.

42. a) Mary says that in order to find the period of the function, she would need to know the coordinates of points $A$ and $E$. Bill says that he could find the period using the coordinates of B and D. Both Mary and Bill are correct. Explain why.
b) Select all the points that represent the x-intercepts of the function.
c) Select all the points that represent the minimum value of the function.
d) Select 2 points that could be used to determine the amplitude of the function.

Explain a process that could be used to determine the amplitude using the 2 selected points.

## Relations and Functions

Graphing Calculator Window Format

$$
\begin{aligned}
& x:\left[x_{\min }, x_{\max }, x_{\mathrm{scl}}\right] \\
& y:\left[y_{\min }, y_{\max }, y_{\mathrm{scl}}\right]
\end{aligned}
$$

Exponents and Logarithms

$$
\begin{aligned}
& y=a^{x} \leftrightarrow x=\log _{a} y \\
& \log _{b} c=\frac{\log _{a} c}{\log _{a} b}
\end{aligned}
$$

Laws of Logarithms

$$
\begin{aligned}
& \log _{a}(M \cdot N)=\log _{a} M+\log _{a} N \\
& \log _{a}\left(\frac{M}{N}\right)=\log _{a} M-\log _{a} N \\
& \log _{a}\left(M^{n}\right)=n \log _{a} M
\end{aligned}
$$

## Logical Reasoning

## Exponential functions

$$
y=a \cdot b^{x}
$$

## Sinusoidal functions

$$
\begin{aligned}
& y=a \cdot \sin (b x+c)+d \\
& \text { Period }=\frac{2 \pi}{b}
\end{aligned}
$$

## Quadratic equations

For $a x^{2}+b x+c=0$

$$
x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}
$$

Probability

$$
n!=n(n-1)(n-2) \ldots 3 \cdot 2 \cdot 1,
$$ where $n \in N$ and $0!=1$

${ }_{n} P_{r}=\frac{n!}{(n-r)!}$
${ }_{n} C_{r}=\frac{n!}{(n-r)!r!}$
${ }_{n} C_{r}=\binom{n}{r}$
$P(A \cup B)=P(A)+P(B)$
$P(A \cup B)=P(A)+P(B)-P(A \cap B)$
$P(A \cap B)=P(A) \cdot P(B)$
$P(A \cap B)=P(A) \cdot P(B \mid A)$
$A^{\prime}$ Complement
$\varnothing$ Empty set
$\cap$ Intersection
$\subset$ Subset
$\cup$ Union

