Math 10-C Polynomials Concept Sheets

Concept 1: Polynomial Intro & Review

A **polynomial** is a mathematical expression with one or more terms in which the exponents are whole numbers and the coefficients are real numbers.

whole number exponents

e.g. x^2 $\qquad \frac{6}{5}y^3 \qquad -5x^2 - x + 3 \qquad 4a^4 + 6ab - 3a^3b^2$

real number coefficients

A **monomial** is a product of a coefficient and one or more variables.

e.g. $6x^2$ variable (x) coefficient $\qquad -3a^3b^2$ coefficient variables (a and b)

Binomial – polynomial with 2 terms

Trinomial – polynomial with 3 terms

Like Terms – terms that have the same variable(s) to the same degree

For example: $3x^2, -4x^2$; $-2x^2y^3,\ 3x^2y^3$

To add and subtract polynomials, collect "like terms".

Example

Simplify the following polynomials.

(a) $4x + 3y - 2x$

$4x + 3y - 2x$

like terms

$= 4x - 2x + 3y$

$= 2x + 3y$

(b) $6a^2 - 4a + 2a^2 + 6a$

$6a^2 - 4a + 2a^2 + 6a$

$= 6a^2 + 2a^2 - 4a + 6a$

$= 8a^2 + 2a$

Concept 2: Multiplying Polynomials

Methods for multiplying large whole numbers without the use of a calculator:

Example:

Multiple 18 x 24 without a calculator.

Method #1: Grid Multiplication

18x24

	10	8
20	200	160
4	40	32

200+160+40+32

= 432

Method #2: Rainbow Method

18x24

$= (10+8)(20+4)$

$= 200+40+160+32$

$= 432$

Examples involving polynomials:

Expand and simplify.

(a) $(c + 3)(c - 7)$

Method 1 : Use a grid diagram.

	c	$+3$
c	c^2	$3c$
-7	$-7c$	21

$= c^2 + 3c - 7c + 21$

$= c^2 - 4c + 21$

Method 2: Use the distributive property.

$$\begin{aligned}(c + 3)(c - 7) &= c(c - 7) + 3(c - 7)\\ &= (c)(c) + (c)(-7) + (3)(c) + (3)(-7)\\ &= c^2 - 7c + 3c - 21 \qquad \text{Combine like terms.}\\ &= c^2 - 4c - 21\end{aligned}$$

(b) $(3k + 4)(k^2 - 2k - 7)$

Method 1 : Use a grid diagram.

	k^2	$-2k$	-7
$3k$	$3k^3$	$-6k^2$	$-21k$
$+4$	$4k^2$	$-8k$	-28

$$= 3k^3 - 6k^2 + 4k^2 - 21k - 8k - 28$$

$$= 3k^3 - 2k^2 - 29k - 28$$

Method 2: Use the distributive property.

Multiply each term in the trinomial by each term in the binomial.

$$\begin{aligned}&(3k + 4)(k^2 - 2k - 7)\\ &= (3k)(k^2 - 2k - 7) + 4(k^2 - 2k - 7)\\ &= (3k)(k^2) + (3k)(-2k) + (3k)(-7) + 4(k^2) + 4(-2k) + 4(-7)\\ &= 3k^3 - 6k^2 - 21k + 4k^2 - 8k - 28\\ &= 3k^3 - 6k^2 + 4k^2 - 21k - 8k - 28 \qquad \text{Combine like terms.}\\ &= 3k^3 - 2k^2 - 29k - 28\end{aligned}$$

(c) $(x+1)(5x+3)+3(2x+4)(6x-2)$

Method 1: Grid Multiplication - Complete in two parts and then combine.

$$3(2x+4)(6x-2)$$
$$=(6x+12)(6x-2)$$

	x	$+1$
$5x$	$5x^2$	$+5x$
$+3$	$3x$	$+3$

$+$

	$6x$	$+12$
$6x$	$36x^2$	$+72x$
-2	$-12x$	-24

$$=5x^2+5x+3x+3 \quad + \quad =36x^2+72x-12x-24$$
$$=5x^2+8x+3 \qquad\qquad =36x^2+60x-24$$

$$\therefore =5x^2+36x^2+8x+60x+3-24$$
$$=41x^2+68x-21$$

Method 2: Distributive Property

$$(x+1)(5x+3)+3(2x+4)(6x-2)$$
$$=(x)(5x+3)+(1)(5x+3)+3[(2x)(6x-2)+(4)(6x-2)]$$
$$=5x^2+3x+5x+3+3(12x^2-4x+24x-8)$$
$$==5x^2+8x+3+3(12x^2+20x-8)$$
$$=5x^2+8x+3+36x^2+60x-24$$
$$=41x^2+68x-21$$

Questions for Practice

Page 87 #2-5, 6 a, c, e, 7 a, c, e, 8 a, c, e, 9-14

Concept 3: Factoring Polynomials

Greatest Common Factor: the largest factor that two or more numbers (or terms) have in common.

Methods for finding the GCF

Determine the greatest common factor of 126 and 144.

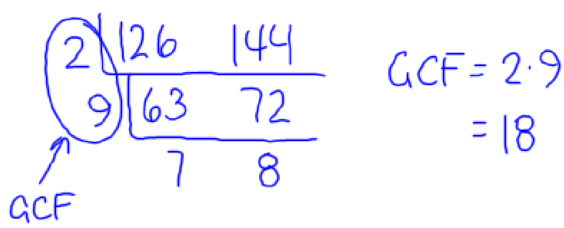
Method #1: List the Factors

126 → 1, 2, 3, 7, 9, 14, 18, 42, 63, 126

144 → 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144

Largest factor that 126 and 144 have in common is 18.

Method #2: Block Method



Method #3: Prime Factorization

Write the prime factorization of each number.
Highlight the factors that appear in each prime factorization.

126 = **2** · **3** · **3** · 7
144 = **2** · 2 · 2 · 2 · **3** · **3**

The greatest common factor is 2 · 3 · 3, which is 18.

Common Multiple: a number that is a multiple of two or more numbers.

Lowest Common Multiple: the smallest number that two or more numbers will divide into evenly

Methods for finding the LCM

Examples:

(a) Determine the least common multiple of 18 and 30.

Method #1: Block Method

$$\begin{array}{r|ll} 2 & 18 & 30 \\ \hline 3 & 9 & 15 \\ \hline & 3 & 5 \end{array} \qquad \begin{aligned} LCM &= 2 \cdot 3 \cdot 3 \cdot 5 \\ &= 90 \end{aligned}$$

Method #2: List the Multiples

18 → 18, 36, 54, 72, 90 108, …

30 → 30, 60, 90 …

<u>GCF Factoring</u>

Greatest common factor (GCF) – A number that divides into every term of polynomial. The GCF can be a monomial or a binomial or a trinomial.

Example:

Find the GCF of the following trinomial:

$20c^4d - 30c^3d^2 - 25cd$

Solution

$20c^4d - 30c^3d^2 - 25cd$ → Factor each term of the trinomial.

$20c^4d = 2 \cdot 2 \cdot \mathbf{5} \cdot c \cdot c \cdot c \cdot \boldsymbol{c} \cdot \boldsymbol{d}$
$30c^3d^2 = 2 \cdot 3 \cdot \mathbf{5} \cdot c \cdot c \cdot \boldsymbol{c} \cdot \boldsymbol{d} \cdot d$
$25cd = \mathbf{5} \cdot 5 \cdot \boldsymbol{c} \cdot \boldsymbol{d}$

The greatest common factor is $5 \cdot c \cdot d = 5cd$

Example:

Factor the following polynomials.

(a) $12a^4 - 9a^2 + 3a$

$= 2 \cdot 2 \cdot 3 \cdot a \cdot a \cdot a \cdot a - 3 \cdot 3 \cdot a \cdot a + 3 \cdot a$

$= 3a(2 \cdot 2 \cdot a \cdot a \cdot a - 3 \cdot a + 1)$

$= 3a(4a^3 - 3a + 1)$

(b) $2x(x-1) - 5(x-1)$

$2x(x-1) - 5(x-1)$

$= (x-1)(2x-5)$

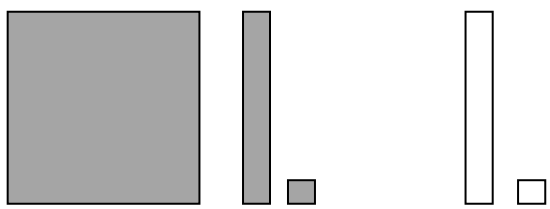
Questions for Practice:

Page 91 #2, 3, 4, 5 d, e, f, 6, 8, 9

<u>Factoring Pictorially</u>

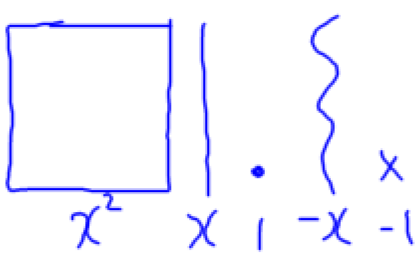
Algebra Tiles

Recall:



Represents: x^2 x 1 $-x$ -1

For simplicity purposes we can sketch them as:



Steps for factoring pictorially:

1. Arrange algebra tiles to make a rectangle. (The coefficient of the x^2 is the number of squares, the coefficient of the x is the number of lines and the constant number is the number of 'dots' or 'x's' to use.)

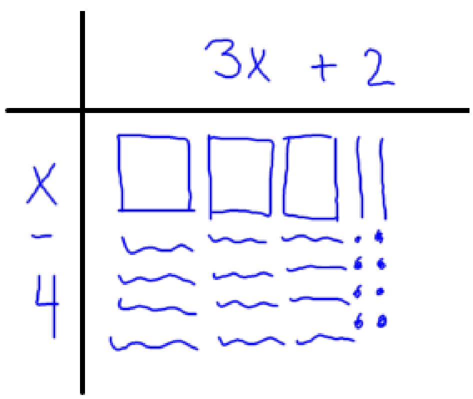
Hint: (Keeping tiles in standard form as shown below will be an advantage)

2. Look at the dimensions of the rectangle and write outside of the grid.

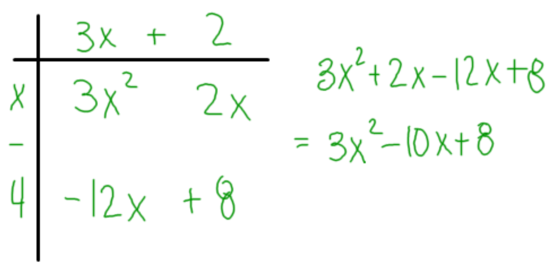
3. The factors are the outside dimensions.

Example:

(a) $3x^2 - 10x + 8$

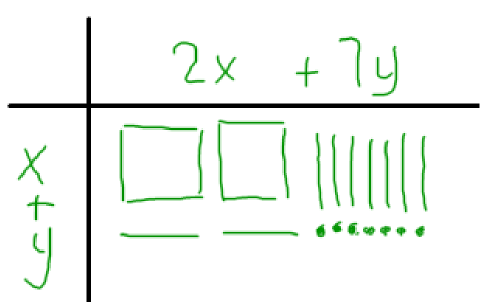


To check your answer, use multiplication:



$\therefore 3x^2 - 10x + 8 = (3x+2)(x-4)$

(b) $2x^2 + 9xy + 7y^2$



$\therefore 2x^2 + 9xy + 7y^2 = (2x+7y)(x+y)$

Questions for Practice:

Page 92 #13; Page 95 #1, 2

Factoring Symbolically

Steps for Factoring Symbolically:

1. Draw the grid.
2. Put the first term in the top left and the last term in the bottom right.
3. Find the product of the coefficients of the first and last term.
4. Find two numbers that have the same product as the first and last term but add up to the coefficient of the middle term.
5. Place the numbers from step 4 in the top right and bottom left position.
6. Take the GCF out of the top row. Put the 'leftover' value above.
7. Repeat for the second row. The factors are on the outside of the grid.

Examples:

Factor:

(a) $8a^2 + 18a - 5$

	$8a^2$	
		-5

Product $= -40$
sum $= 18$
#'s $-2, 20$

$\rightarrow$

$8a^2 \div 2a$ $\quad$ $-2a \div 2a$

	$4a$	-1
$+2a$	GCF $\leftarrow 8a^2$	$-2a$
5	GCF $\leftarrow 20a$	-5

(b) $2x^2 + 9xy + 7y^2$

	$2x^2$	
		$7y^2$

Prod $= 14$
sum $= 9$
#'s: 2 and 7

	x	y
$2x$	$\leftarrow 2x^2$	$2xy$
$7y$	$7xy$	$7y^2$

Finding Missing Values in Trinomials:

Example:

Determine the values of *b* that allow the expression $3y^2 + ny + 16$ to be factored.

	$3y^2$	___
	___	16

prod = 48
sum = ?
Options:
1, 48 → sum = 49
2, 24 → sum = 26
3, 16 → sum = 19
etc...

Therefore, *b* could be 49 or 26 or 19 or …

Questions for Practice:
Page 95 #4-7 (Factoring Trinomials); Page 96 #8-11 (Finding Missing Values);
Page 96 #12-19 (Word Problems)

<u>Factoring Special Cases</u>

Perfect Square Trinomials

The trinomial $x^2 + 4x + 4$ is a perfect square
The first term is a perfect square: x^2
The last term is a perfect square: 2^2
The middle term is twice the product of the square root of the first term and the square root of the last term: (2)(*x*)(2) = 4*x*

Examples:

Note: Always check to make sure it's a perfect square trinomial first!

(a) $x^2 + 10x + 25$

Check: $\sqrt{x^2} = x$ and $\sqrt{25} = 5$ and $2 \times 5 = 10$ **OR**

so
$$= (x+5)(x+5)$$
$$= (x+5)^2$$

	x	5
x	x^2	5x
5	5x	25

prod = 25
sum = 10
#'s 5, 5
∴ (x+5)(x+5)

(b) $a^2 - 12ab + 36b^2$

Check: $\sqrt{a^2} = a$ and $\sqrt{36b^2} = 6b$ and $6 \times 2 = 12$

so
$$= (a - 6b)(a - 6b)$$
$$= (a - 6b)^2$$

Difference of Squares

Difference of Squares – a polynomial expressed in the form $x^2 - y^2$

- results when you multiply two binomials that are the sum and the difference of the same two quantities

Example:

Note: Always check for a GCF first…

(a) $2k^2 - 18$
$= 2(k^2 - 9)$
$= 2(k + 3)(k - 3)$

OR

$= 2(k^2 - 9)$
$= 2(k^2 + 0k - 9)$

	k	$+3$
k	k^2	$+3k$
-3	$-3k$	-9

Product $= -9$
Sum $= 0$
#'s: $+3, -3$

$\therefore 2k^2 - 18 = 2(k+3)(k-3)$

(b) $81 - a^4$
$= (9 + a^2)(9 - a^2)$
$= (9 + a^2)(3 + a)(3 - a)$

Questions for Practice:

Page 99 #1, 2a,c,e, 3 a,c,e, 4, 5-7, 8-10, 13-16

<u>Unit Review Questions</u>

Page 102 #1-18