Math 10 C: Relations & Functions

Concept #1-3: Intro to Graphing, Graphing Relations & Interpreting Graphs

Vocabulary and Key Concepts

Independent Variables - the variable for which values are selected (the input)

Dependent Variables - the variable whose values depend on those of the independent variable (the output)

Discrete Data - data values on a graph that are not connected

Continuous Data - data values on a graph that are connected

Restrictions:

Domain: the set of all possible values for the independent variable (all possible inputs)
Range: the set of all possible values for the dependent variable (all possible outputs)

Linear Relations: a relationship between two variables where the independent and the dependent variable change at a constant rate (not necessarily the same rate for independent and dependent).
Ex) The total cost of a banquet, *C*, depends on the number of people, *n* attending.

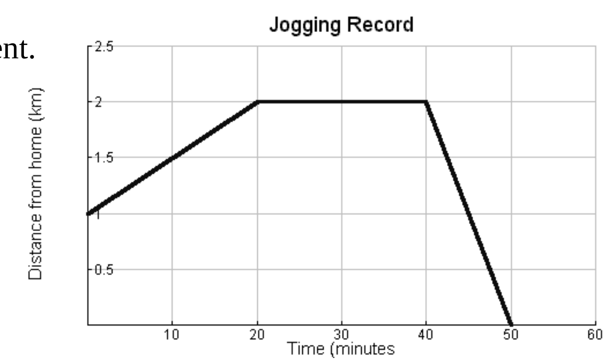
Non-Linear Relations: a relationship between two variables that is not linear.

Skills

Describing Graphs with Words

Example #1

Tell the story that the graph at right could represent.



Answers will vary…

A man leaves a store that is 1 km from his home and walks to his friends house 2 km from his own at a steady rate, which takes him 20 minutes. He stops for a 20 minute visit, and then runs home at a faster pace. It takes him 10 minutes to get the 2 km home.

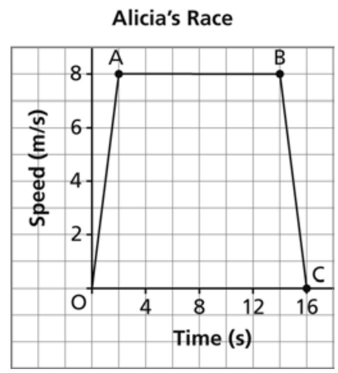
Sketching a Graph from Words

Example #2

At the beginning of a race, Alicia took 2 s to reach a speed of 8 m/s. She ran at approximately 8 m/s for 12 s, then slowed down to a stop in 2 s. Sketch a graph of speed as a function of time. Label each section of your graph, and explain what it represents.

Solution

Draw and label axes on a grid. The horizontal axis represents time in seconds. The vertical axis represents speed in metres per second.



Segment	Journey
OA	Alicia's speed increases from 0 to 8 m/s, so the segment goes up to the right for the first 2 s.
AB	Alicia runs at approximately 8 m/s for 12 s. Her speed does not change, so the segment is horizontal.
BC	Alicia slows down to 0 km/h in 2 s, so her speed decreases and the segment goes down to the right.

Representing Data

There are 5 ways we represent data:

1. Words
2. Table of Values
3. Equation
4. Ordered Pairs
5. Graph

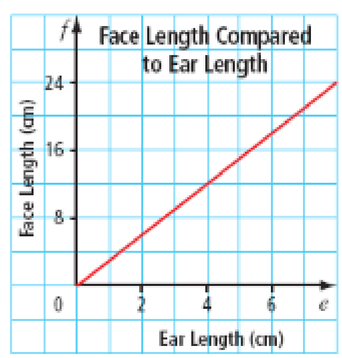
Example #3

Words
Three times the length of your ear, *e*, is equal to the length of your face, *f*. (from chin to hairline).

Equation

$f = 3e$

Graph



Ordered Pairs
(4, 12), (4.5, 13.5), (5, 15), (5.5, 16.5), (6, 18), (6.5, 19.5)

Table of Values

Ear Length, *e* (cm)	Face Length, *f* (cm)
4	12
4.5	13.5
5	15
5.5	16.5
6	18
6.5	19.5

Answering Questions About a Scenario

The table of values shows the cost of movie tickets at a local theatre. The maximum number of people that can fit in the theatre is 300.

Number of Tickets	**Cost ($)**
1	12
2	24
3	36
4	48

(a) Is this a linear or non-linear relationship? Explain how you know.

Linear – The cost increases at the same rate for every increase in the number of tickets.

(b) Assign a variable to represent each quantity in the relation. Which variable is the dependent variable and which is the independent variable?

Number of tickets – n – independent variable
Cost – C – dependent variable

(c) Are the data discrete or continuous? Explain how you know

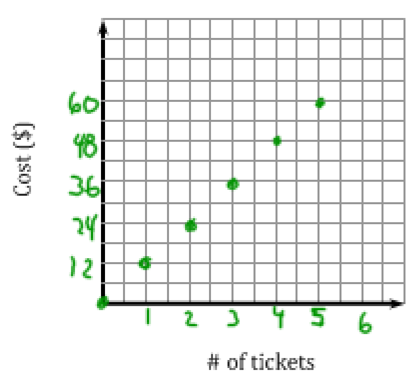
Discrete – cannot buy a portion of a ticket.

(d) Are there restrictions on the variables? What is the domain and range?

Number of tickets sold must be between 0 and 300 (max capacity of theatre) and must be a whole number.

Cost must be positive and will be a multiple of 12.

(e) Graph the data.

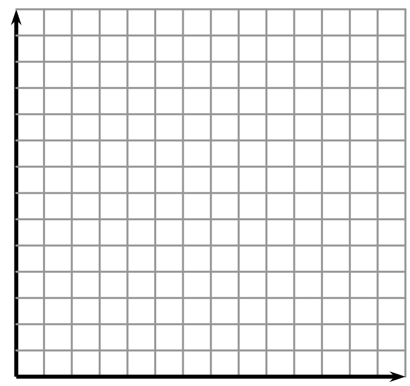


Example #4

A 5L juice jug weighs 0.3 kg. Each litre of orange juice has a mass of 2.1 kg. Complete the table below to show the mass of the jug with each amount of juice.

(a) Graph this function on the grid. Include a title and labels for the axes.

Litres of orange juice (L)	Total mass of jug (kg)
0	*0.3*
1	*2.4*
2	*6.5*
3	*8.6*
4	*10.7*
5	*12.8*



(b) Write an equation that represents this relation.

M = mass of jug
l = litres of juice

$M = 0.3 + 2.1\,l$

(c) Is the data continuous or discrete? Explain why or why not.

Continuous - we can have a portion of a litre of juice.

(d) What is the domain and range of the graph?

Domain – must be greater than or equal to zero and less than or equal to 5
Range – mass must be greater than 0.3 and less than or equal to 12.8 kg

Concept #4: Domain and Range

Vocabulary and Key Concepts

Domain

- the set of all possible input values (x-values) for a relation
- distance graph goes left and right
- set of first coordinates of the ordered pairs

Range

- the set of all possible output values (y-values) for a relation
- distance graph goes up and down
- set of second coordinates of the ordered pairs

Skills

Stating Domain and Range in Words

Example #1

The total amount of money paid for a play is determined by the equation $A = 20n$, where n is the number of people in attendance. The theatre holds 250 people.

(a) State the independent and dependent variable.

Independent – number of people
Dependent – Total money paid

(b) What are the domain and range of this function?

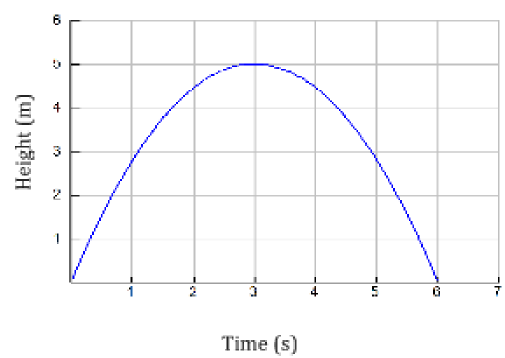
The domain is all integers between 0 and 250 inclusive
The range is all multiples of 20 between 0 and 5000 inclusive.

Example #2

The graph below represents the height of a soccer ball that is kicked into the air.

What are the domain and range of this function?

Domain – all real numbers between 0 and 6 inclusive
Range – all real numbers between 0 and 5 inclusive



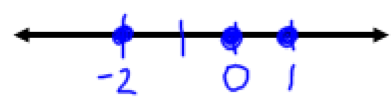
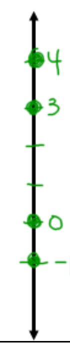
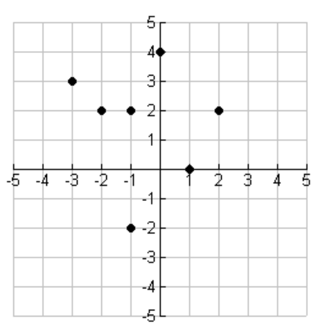
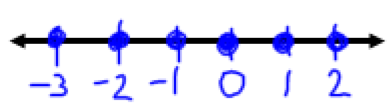
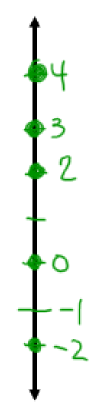
__Stating the Domain and Range of Discrete Relations__

The Domain and Range of discrete relations can be stated using:

- Words
- A list
- A number line

Example #3

State the domain and range for the following relations:

		Domain	Range
(a) {(-2, 0), (-2, 4), (0, 3), (1, -1)}	**Words**	x is equal to -2, 0 and 1	y is equal to -1, 0, 3 and 4
	A list	{-2, 0, 1}	{-1, 0, 3, 4}
	A number line		
(b) 	**Words**	x is equal to -3, -2, -1, 0, 1, and 2	y is equal to -2, 0, 2, 3, and 4
	A list	{-3, -2, -1, 0, 1, 2}	{-2, 0, 2, 3, 4}
	A number line		

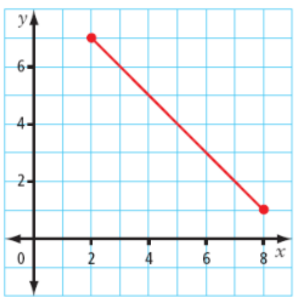
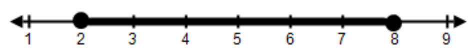
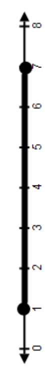
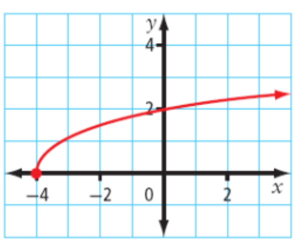
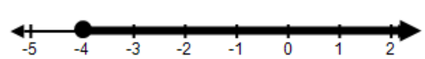
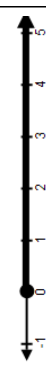
Stating the Domain and Range of Continuous Relations

The Domain and Range of Continuous relations can be stated as:

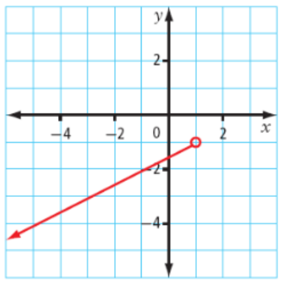
- Words
- Number Line
- Set Notation
- Interval Notation

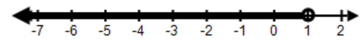
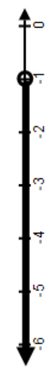
Example #4

Express the domain and range of each graph. Use words, a number line, interval notation and set notation.

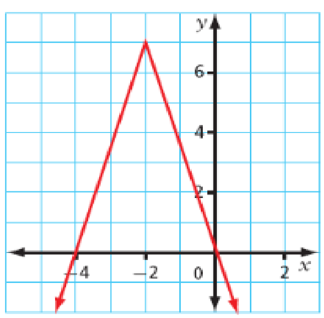
		Domain	Range
(a) 	**Words**	x is all real numbers between 2 and 8 inclusive	y is all real numbers between 1 and 7 inclusive
	Number Line		
	Set Notation	$\{x \mid 2 \le x \le 8, x \in R\}$	$\{y \mid 1 \le y \le 7, y \in R\}$
	Interval Notation	[2, 8]	[1, 7]
(b) 	**Words**	x is all real numbers greater than or equal to -4	y is all real numbers greater than or equal to 0
	Number Line		
	Set Notation	$\{x \mid x \ge -4, x \in R\}$	$\{y \mid y \ge 0, y \in R\}$
	Interval Notation	$[-4, \infty)$	$[0, \infty)$

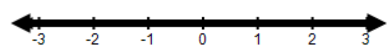
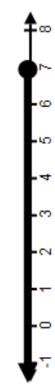
(c)



Words	x is all real numbers less than 1	y is all real numbers less than -1
Number Line		
Set Notation	$\{x \mid x < 1, x \in R\}$	$\{y \mid y < -1, y \in R\}$
Interval Notation	$(-\infty, 1)$	$(-\infty, -1)$

(d)



Words	x is all real numbers	y is all real numbers less than or equal to 7
Number Line		
Set Notation	$\{x \mid x \in R\}$	$\{y \mid y \le 7, y \in R\}$
Interval Notation	$(-\infty, \infty)$	$(-\infty, 7)$

Concept #5: Functions

Vocabulary and Key Concepts

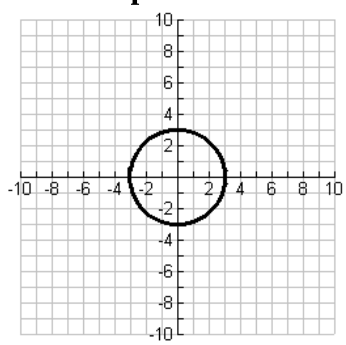
Relation

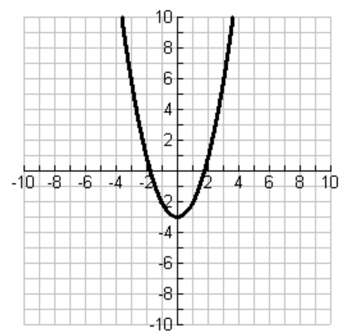
- a rule that gives one or more output numbers (y values) for every valid input number (x value).
- any graph, any equation or any set of ordered pairs

Function

- a rule that gives a single output number (y value) for every valid input number (x value).
- a special type of relation where every x in the ordered pairs are different
- Satisfies the **vertical line test** – if a vertical line can be drawn through any part of the graph without touching more than one point, the relation is a function
- In general, if a value of x is repeated, it is not a function.

For example:





***Not a function** – inputs have more than one output (does not pass the 'vertical line test')*
For example – if input (x) is 0, output (y) is 3 and -3

***Function** – every input has only one output (passes the 'vertical line test')*

Skills

Example #1

The rule is to "square a number."
The input numbers are $-4, -3, -2, -1,\ 0,\ 1,\ 2,\ 3,\ 4$ (or $-4 \le x \le 4,\ x\ \varepsilon\ I$)

(a) Determine the output numbers.
Output → 16, 9, 4, 1, 0, 1, 4, 9, 16

(b) Is this rule a function? Explain.
It is a function, every input has only one output (although the outputs are repeated)

Example #2

Classify the following as functions or relations. Explain your answer.

(a) {(2, 3), (4, -1), (6, 3), (8, -1)}

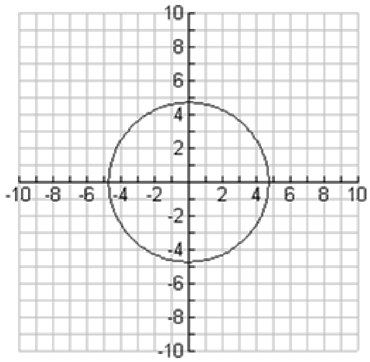
Function – no values of x have more than one output.

(b)

x	1	1	3	5	7
y	2	4	6	8	10

Not a function – if x=1, y has two different answers. (Graph would not pass the vertical line test)

(c)



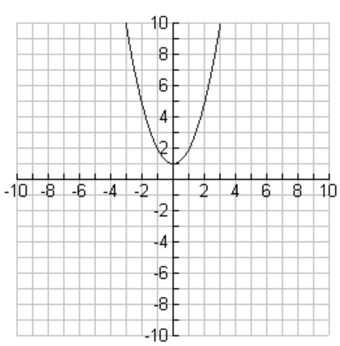
Not a function – does not pass the vertical line test.

<u>Functional Notation</u>

Define: $y = f(x) = f$ of x ➔ the value of a function at a given value of x

Example #1

Find the value of y at the given value of x in the following graph.



$f(2) = 5$

$f(-3) = 10$

$f(0) = 1$

If the equation for the graph at right is $f(x) = x^2 + 1$, calculate $f(2)$ **algebraically** by substituting 2 in for x.

Example #2

The height of a ball thrown up into the air from ground level is a function of time, as given by the equation $h(t) = -4t^2 + 16t$, where t is the time in seconds and h is the height in meters.

(a) Explain in words what $h(2) = 16$ means in this situation.

The height of the ball when time is 2 seconds is 16 meters

(b) How high would the ball be after 3 seconds? Express your answer in function notation.

$$\begin{aligned} h(t) &= -4t^2 + 16t \\ h(3) &= -4(3)^2 + 16(3) \\ h(3) &= -4(9) + 48 \\ h(3) &= -36 + 48 \\ h(3) &= 12 \end{aligned}$$

Therefore, the height of the ball when time is 3 seconds is 12 m.

Example #3

If $f(x) = x^2 - 3x + 7$, calculate:

(a) $f(2)$

$$\begin{aligned} f(2) &= (2)^2 - 3(2) + 7 \\ f(2) &= 4 - 6 + 7 \\ f(2) &= -2 + 7 \\ f(2) &= 5 \end{aligned}$$

(b) $f(-1)$

$$\begin{aligned} f(-1) &= (-1)^2 - 3(-1) + 7 \\ f(-1) &= 1 + 3 + 7 \\ f(-1) &= 11 \end{aligned}$$

Example #4

If $g(x) = 7 + 2x$, calculate $g(x) = -5$

$$\begin{aligned} -5 &= 7 + 2x \\ -5 - 7 &= 2x \\ -12 &= 2x \\ x &= -6 \end{aligned}$$