Math 10 C: Linear Equations & Graphs

Slope-Intercept Form

Note*: For all graphs shown, if the line goes to the end of the grid, assume that the domain and ranges are all real numbers.*

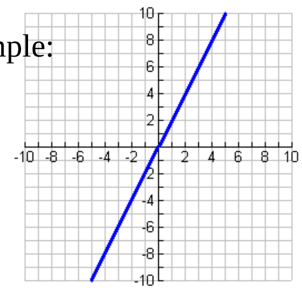
Vocabulary and Key Concepts

Slope – the ratio of the vertical change (rise), to the horizontal change (run) of a line or line segment

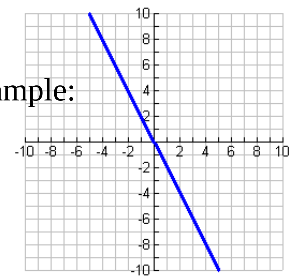
$$\text{Slope } (m) = \frac{change\ in\ y}{change\ in\ x} \quad \text{OR} \quad m = \frac{y_2 - y_1}{x_2 - x_1}$$

If the slope of a line segment is:

- **positive**, then the line segment rises from left to right. For Example:

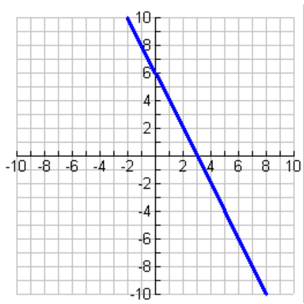


- **negative**, then the line segment falls from left to right. For Example:



***y*-intercept –** the *y*-coordinate of the point where a line or curve crosses the *y*-axis. The value of *x* is always 0 at the *y* intercept.

For Example:



y-intercept is 6 (a point on the line is (0,6)

Slope-intercept form – the equation of a line in the form $y = mx + b$, where *m* is the slope of the line and *b* is the *y*-intercept of the line.

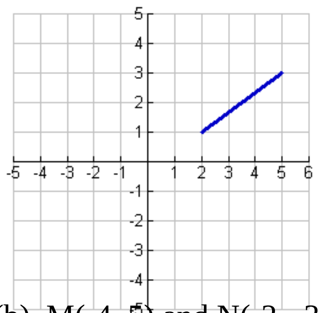
<u>Skills</u>

Determining the Slope of a Line Segment

Example #1

Determine the slope of the line segment that joins:

(a) A(2, 1) and B(5, 3)



From the graph: $m = \dfrac{rise}{run}$

Count: Rise: up 2 (+2)
Run: right 3 (+3)

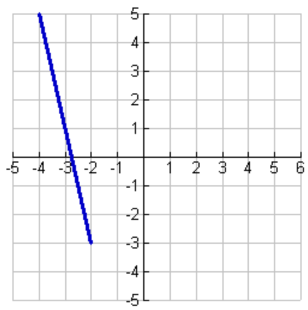
$$m = \frac{2}{3}$$

From equation:

$$m = \frac{3-1}{5-2}$$

$$m = \frac{2}{3}$$

(b) M(-4, 5) and N(-2, -3)



From the graph: $m = \dfrac{rise}{run}$

Count: Rise: down 8 (-8) OR Rise: up 8 (+8)
Run: right 2 (+2) Run: left 2 (-2)

$$m = \frac{-8}{2} \qquad m = \frac{8}{-2}$$

$$m = -4 \qquad m = -4$$

From equation:

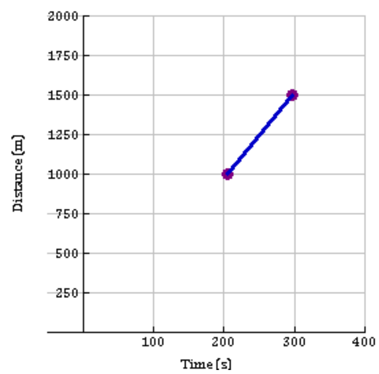
$$m = \frac{-3-5}{-2--4}$$

$$m = \frac{-8}{2}$$

$$m = -4$$

Example #2

The graph shows the approximate times at the 1000 m mark and at the 1500 m mark for a rowing crew of the girls' junior open eight race at the Brentwood Regatta. Determine the average rate of change for this portion of the race.



$$m = \frac{1500m - 1000m}{297s - 205s}$$

$$= \frac{500m}{92s}$$

$$= \frac{125}{23} m/s$$

Finding other Points on a line given Slope and 1 Point

Example #3

A line segment has a slope of $-\frac{2}{3}$. One point on the line is (-2, -4). Find 2 other points on the line.

Notice: Slope can be written as $\frac{-2}{3}$ *or* $\frac{2}{-3}$.

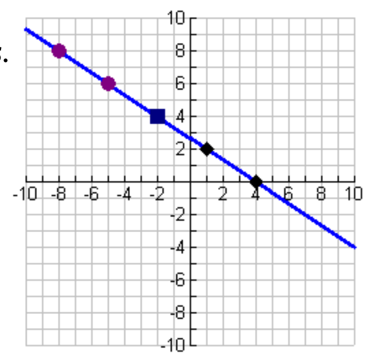
Steps:

1. Plot the point given (-2, 4) *(shown by the square)*
2. Count down 2, then right 3 for the next point (-1, 2) *(shown by the diamond).*
3. Repeat for the next point (4, 0) (or (-8, 8)) *(shown by the diamond).*

Or

1. Plot the point given (-2, 4) *(shown by the square)*
2. Count up 2, left 3 to (-5, 6) *(shown by the circles)*
3. Repeat for the next point (-8, 8) *(shown by the circles).*

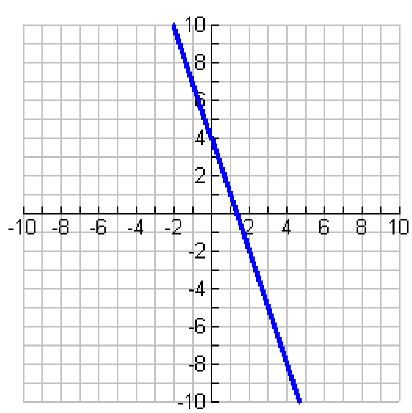
Note: All of the points shown are solutions.



Finding the Slope of a Line from a Graph

Example #4

Find the slope of the line shown below.



Find 2 'nice' points:
For example: (0, 4) and (2, -2)
Count rise and run the same as Example #1 with line segments, or use the slope formula.

$$m = \frac{-2-4}{2-0}$$

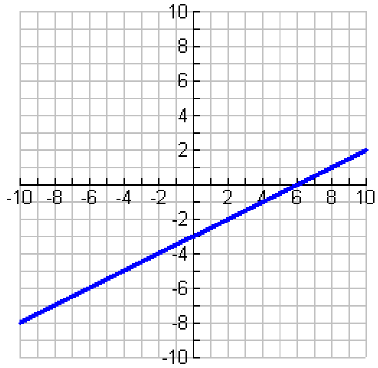
$$m = \frac{-6}{2}$$

$$m = -3$$

Writing the Equation of a Line ($y = mx + b$***)***

Example #5

Write the equation of the line below.



Steps:

1. Look at the *y*-intercept of the graph. If the *y*-intercept is an integer,
   then you know the value of *b*.
   $b = -3$
2. Find another point that crosses at integer values.
   (2, -2)
3. Count out slope to that point.
   Up 1, right 2
4. State the slope
   $m = \frac{1}{2}$
5. State the equation of the line.
   $y = \frac{1}{2}x - 3$

Graphing Equations in the form $y = mx + b$

Steps for graphing equations

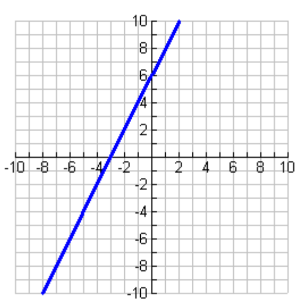
1. Plot the *y*-intercept (*b*).
2. Starting from the *y*-intercept, count the slope (*m*) - rise over run.
3. Connect the points.
4. Label the line with the equation.

Example #6

For the following lines, state the *y* intercept and slope, then graph.

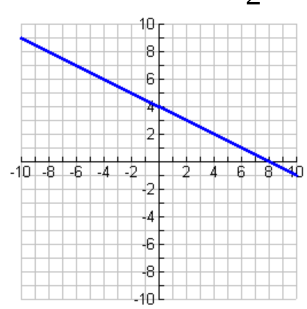
(a) $y = 2x + 6$

$m = 2$ $b = 6$



(b) $y = \frac{-1}{2}x + 4$

$m = \frac{-1}{2}$ $b = 4$



General Form of a Line

Vocabulary and Key Concepts

***x*-intercept** – the *x*-coordinate where the graph crosses the *x*-axis

- the value of *y* at the *x*-intercept is always 0

***y*-intercept – the *y*-coordinate where the graph crosses the *y*-axis**

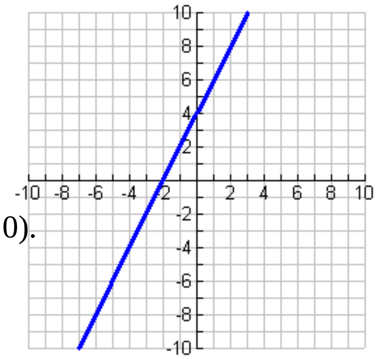
-the value of *x* at the *y*-intercept is always 0

For example:

In the graph to the right:

The *x*-intercept is -2 and the *y*-intercept is 4 .

Therefore, two coordinates on the graph are (0, 4) and (-2, 0).



General Form Equation – The equation of a line written in the form $Ax + By + C = 0$, where A, B, and C are integers and *A* and *B* are not both 0. In general, the value of *A* should be positive. For example $2x - 3y + 6 = 0$

Skills

Horizontal and Vertical Lines

Example #1
Graph the line with each equation on the grid.

(a) $2y + 4 = 0$

$$2y = -4$$

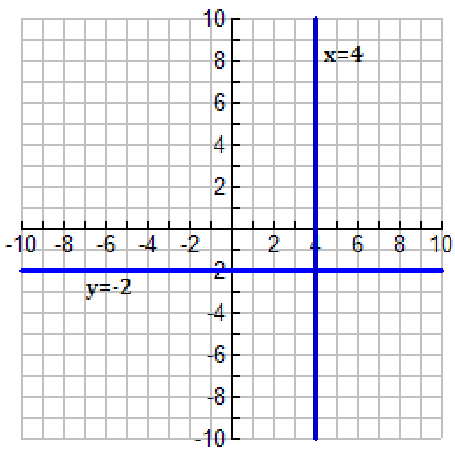
$$y = \frac{-4}{2} \quad \text{(slope is 0)}$$

$$y = -2$$

(b) $3x - 12 = 0$

$$3x = 12$$

$$x = 4 \quad \text{(slope is undefined)}$$



Finding x and y-intercepts to Graph an Equation

Example #2

Find the x and y-intercepts to graph the line represented by each equation. Then, write the equation of the line in slope-intercept form.

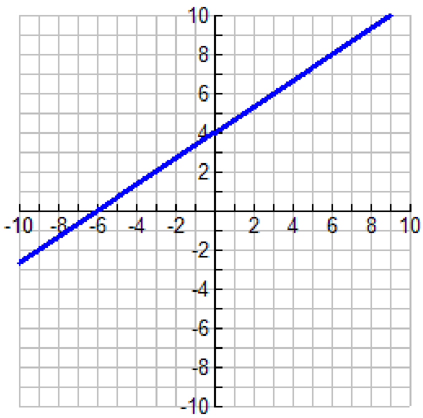
(a) $2x-3y+12=0$

x-int (y=0)

$$2x-3(0)+12=0$$
$$2x+12=0$$
$$2x=-12$$
$$x=-6$$

y-int (x=0)

$$2(0)-3y+12=0$$
$$-3y+12=0$$
$$-3y=-12$$
$$y=4$$



$$m=\frac{4}{6}$$
$$m=\frac{2}{3}$$

b = 4

$$y=\frac{2}{3}x+4$$

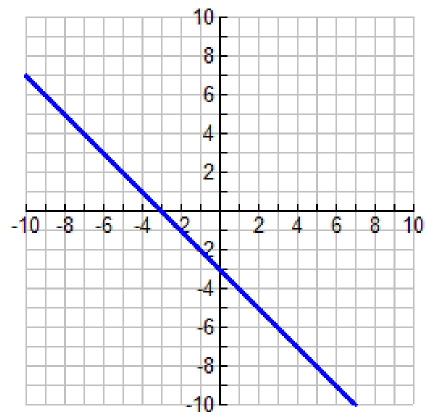
(b) $-3y=3x+9$

x-int (y=0)

$$-3(0)=3x+9$$
$$-9=3x$$
$$x=-3$$

y-int (x=0)

$$-3y=3(0)+9$$
$$-3y=9$$
$$y=-3$$



$$m=\frac{-3}{3}$$
$$m=-1$$

b = -3

$$y=-x-3$$

Re-writing the Equation of a Line in Different Forms

Convert the following equations from slope y-intercept form to general form.

Steps:
1. If there are fractions, multiply every term by the lowest common multiple of the denominators.
2. Cancel out common factors of the numerator and the denominator.
3. Move all terms to one side of the equal sign (the number in front of the x should be positive)

Example #3

Convert the following equations from slope-intercept form to general form.

(a) $y = \frac{3}{2}x - \frac{3}{2}$

$$2y = (\cancel{2})\frac{3}{\cancel{2}}x - (\cancel{2})\frac{3}{\cancel{2}}$$
$$2y = 3x - 3$$
$$0 = 3x - 2y - 3$$

(b) $y = -2x + \frac{1}{4}$

$$4y = (4) - 2x + (\cancel{4})\frac{1}{\cancel{4}}$$
$$4y = 4 - 2x$$
$$0 = 2x + 4y - 4$$

Example #4

Convert the following equations from general form to slope y-intercept form then state the slope and y-intercept of the line.

(a) $3x + 5y - 6 = 0$

$$5y = -3x + 6$$
$$y = \frac{-3}{5}x + \frac{6}{5}$$

$$m = \frac{-3}{5} \quad b = \frac{6}{5}$$

(b) $2x - y - 4 = 0$

$$2x - y - 4 = 0$$
$$-y = -2x + 4$$
$$y = 2x - 4$$

$$m = 2 \quad b = -4$$

Example #5

Given the formula $y = mx + 5$, find the value of m if the line goes through the point (2, -3).

$$-3 = m(2) + 5$$
$$x = 2; y = -3 \rightarrow \quad -3 - 5 = 2m$$
$$-8 = 2m$$
$$m = -4$$

Example #6

A line has a slope of $\frac{2}{3}$ and goes through the point (-4, 0). Determine the *y*-intercept of the line, then write the equation of the line in slope intercept **and** general form.

$$m=\frac{2}{3},\ \ x=-4,\ \ y=0$$
$$y=mx+b$$
$$0=\frac{2}{3}(-4)+b$$
$$0=\frac{-8}{3}+b$$
$$b=\frac{8}{3}$$

Therefore:

Slope Intercept form: $y=\frac{2}{3}x+\frac{8}{3}$

$$3y=(\cancel{3})\frac{2}{\cancel{3}}x+(\cancel{3})\frac{8}{\cancel{3}}$$
$$3y=2x+8$$

General form: $0=2x-3y+8$

Writing the Equation of a Line

Vocabulary and Key Concepts

Finding the Equation of a Line

To find the equation of a line, you must determine the slope of the line and a point on the line.

Recall: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ so $y_2 - y_1 = m(x_2 - x_1)$

Slope Point Form – Equation of a line in the form $y - y_1 = m(x - x_1)$ where (x_1, y_1) is a point on the line and m is the slope of the line.

Skills

Writing the Equation of a Line Given the Slope and a Point using Substitution

Steps:

1. Start with $y = mx + b$.
2. Substitute the slope of the line for *m*.
3. Substitute the point given (x, y) for *x* and *y* in the slope intercept equation.
4. Solve for *b*.
5. Write the equation of the line with the slope given and the *b* value calculated.
6. Re-write in general form if asked for in the question.

Example #2
Find the equation of the line that passes through (1, 4) with a slope of 3. Write your answer in slope-intercept and General form.

$y = mx + b$

$y = 3x + b$

$4 = 3(1) + b$

$4 - 3 = b$

$b = 1$

$y = mx + b$

Slope intercept form: $y = 3x + 1$

$y = 3x + 1$

General Form: $0 = 3x - y + 1$

Writing the Equation of a Line Given the Slope and a Point using Slope Point Method

Steps:

1. Start with $y - y_1 = m(x - x_1)$.
2. Substitute the slope of the line for *m* and the point given for x_1 and y_1.
3. Multiply the slope into the brackets.
4. Re-arrange the equation into the form you are asked for. (If the question asks for both slope-intercept and general form – do either one first and then convert the equation)

Example #2

Find the equation of the line that passes through (1, 4) with a slope of 3. Leave your answer in standard form.

$$y - y_1 = m(x - x_1)$$
$$y - 4 = 3(x - 1)$$
$$y - 4 = 3(x - 1)$$
$$y = 3x + 1$$
$$y - 4 = 3x - 3$$

General Form: $0 = 3x - y + 1$

$$y = 3x - 3 + 4$$

Slope Intercept Form: $y = 3x + 1$

Writing the Equation of a Line Given 2 Points using Substitution

Steps:

1. Start with $y = mx + b$.
2. Use the slope formula and the two points given to determine the slope of the line.
3. Substitute the slope of the line for *m*.
4. Substitute one of the points given (x, y) for *x* and *y* in the slope intercept equation. (The answer will be the same regardless of the point you choose.
5. Solve for *b*.
6. Write the equation of the line with the slope given and the *b* value calculated.
7. Re-write in general form if asked for in the question.

Example #3

Find the equation of the line that passes through (-1, 2) and (4, 5). Leave your answer in slope-intercept **and** general form.

$$m = \frac{5-2}{4--1}$$

$$m = \frac{3}{5}$$

$$y = mx + b$$

$$5 = \frac{3}{5}(4) + b$$

$$5 = \frac{12}{5} + b$$

$$b = 5 - \frac{12}{5}$$

$$b = \frac{25}{5} - \frac{12}{5}$$

$$b = \frac{13}{5}$$

$$y = mx + b$$

Slope –intercept Form $y = \frac{3}{5}x + \frac{13}{5}$

$$5y = 3x + 13$$

General Form $0 = 3x - 5y + 13$

Writing the Equation of a Line Given 2 Points using Slope Point Method

Steps

1. Start with $y - y_1 = m(x - x_1)$.
2. Use the slope formula and the two points given to determine the slope of the line.
3. Substitute the slope of the line for *m* and the point given for x_1 and y_1.
4. Multiply the slope into the brackets.
5. Re-arrange the equation into the form you are asked for. (If the question asks for both slope-intercept and general form – do either one first and then convert the equation)

Example #4

Find the equation of the line that passes through (-4, 3) and (3, -1). Leave your answer in slope-intercept **and** general form.

$$m = \frac{-1-3}{3--4}$$

$$m = \frac{-4}{7}$$

$$y - y_1 = m(x - x_1)$$

$$y - -1 = \frac{-4}{7}(x-3)$$

Slope-Intercept Form

$$y + 1 = \frac{-4}{7}x + \frac{12}{7}$$

$$y = \frac{-4}{7}x + \frac{12}{7} - 1$$

$$y = -\frac{4}{7}x + \frac{12}{7} - \frac{7}{7}$$

$$y = \frac{-4}{7}x + \frac{5}{7}$$

General Form

$$7y + 1(7) = (7)\frac{-4}{7}(x-3)$$

$$7y + 7 = -4(x-3)$$

$$7y + 7 = -4x + 12$$

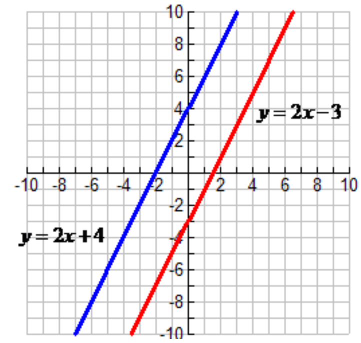
$$4x + 7y - 5 = 0$$

Parallel and Perpendicular Lines

Vocabulary and Key Concepts

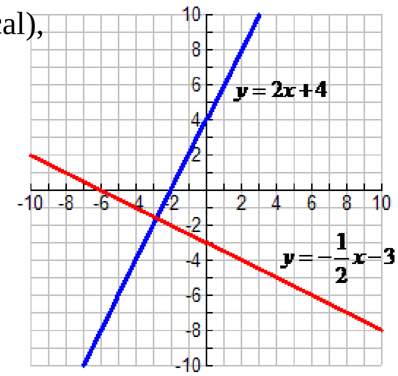
Parallel Lines:

- If two non-vertical line segments are parallel, their slopes are equal.
- If the slopes of two line segments are equal, the segments are parallel.



Perpendicular Lines

- If two line segments are perpendicular (and neither one is vertical), their slopes are negative reciprocals.
- If the slopes of two line segments are negative reciprocals, the segments are perpendicular.



Recall: The numbers $\frac{3}{5}$ and $\frac{-5}{3}$ are *negative reciprocals*.

(Reciprocals with the opposite signs.)

The product of negative reciprocals is –1.

Example: $\frac{3}{5} \times \frac{-5}{3} = \frac{-15}{15} = -1$

Skills

Write the Equation of Lines that are Parallel or Perpendicular

Example #1

Find the equation of a line, L_1, that has a x-intercept of 4 and is parallel to the line, L_2, $0 = 3x - 2y + 2$. Leave your answer in slope intercept form **and** general form.

$L_2 \rightarrow$ $0 = 3x - 2y + 2$

$2y = 3x + 2$

$y = \frac{3}{2}x + 1$

$y = \frac{3}{2}x - 6$

$m_{L_1} = \frac{3}{2} \rightarrow m_{L_2} = \frac{3}{2}$

$2y = 3x - 12$

$0 = 3x - 2y - 12$

$L_1 \rightarrow$ $m = \frac{3}{2}$ through (4, 0)

$y = \frac{3}{2}x + b$

$0 = \frac{3}{2}(4) + b$

$0 = \frac{12}{2} + b$

$-6 = b$

Therefore, $L_1 \rightarrow$

Slope-intercept form:

General Form:

Example #2

Find the equation of a line, L_1, that goes through (-1, 4) and is perpendicular to the line, L_2 with equation $3x-4y+12=0$. Leave your answer in slope intercept form.

$\boldsymbol{L_2} \rightarrow$ $3x-4y+12=0$

$$-4y=-3x-12$$

$$y=\frac{3}{4}x+3$$

$$m=\frac{3}{4}$$

$$m_{\perp}=-\frac{4}{3}$$

$\boldsymbol{L_1} \rightarrow$ $m=-\frac{4}{3}$ through (-1, 4)

$$y=-\frac{4}{3}x+b$$

$$4=-\frac{4}{3}(-1)+b$$

$$4=\frac{4}{3}+b$$

$$\frac{12}{3}-\frac{4}{3}=b$$

$$b=\frac{8}{3}$$

Therefore, $\boldsymbol{L_1}$ $\rightarrow$ $y=-\frac{4}{3}x+\frac{8}{3}$