Math 10 C: Systems of Equations

Concept Sheet #1&#2: Solving Linear Systems Graphically & Number of Solutions

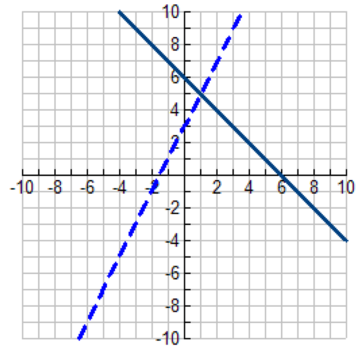
Vocabulary & Key Concepts:

System of Linear Equations: two or more equations of linear functions in the same variables.

Solution: of a system of linear equations is the pair of values that satisfy both equations. Often, we graph the equations on a coordinate plan in terms of *x* and *y* – the solution is the point of intersection of the two lines.

For example:
Given the system of equations at right:
$y = 2x + 3$ and $y = -x + 6$

The solution is (1, 5).
In other words, when *x* = 1 and *y* = 5, the equations are both satisfied (true).



Determining if a point is a "solution" to a system of equations.

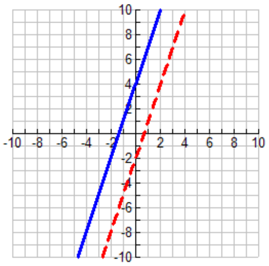
- ➔ If ordered pairs are substituted into an equation and make the equation TRUE, then we say that the ordered pair "satisfies" the equation.
- ➔ If an ordered pair satisfies both equations, that means it is the point of intersection between the two lines and therefore it is the solution to the system.

Number of Solutions in a Linear System of Equations

Three cases:

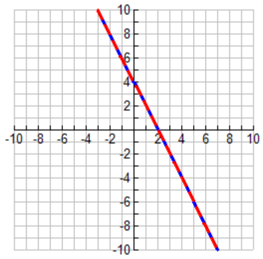
1. No Solutions - Parallel lines (same slope, different *y*-intercepts)
2. Infinite Solutions – Same line (same slope and same *y*-intercept)
3. One Solution – Lines with different slopes

Case 1



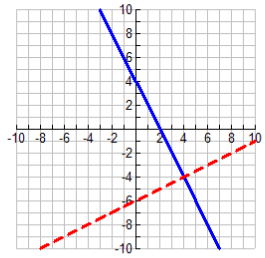
$$y = 3x + 4$$
$$y = 3x - 2$$

Case 2



$$y = -2x + 4$$
$$2y = -4x + 8$$

Case 3



$$y = -2x + 4$$
$$y = \frac{1}{2}x - 6$$

Skills:

Example #1

A system is defined by the equations:

$$\text{Equation 1} \rightarrow \; 2x + y = 5$$
$$\text{Equation 2} \rightarrow \; x + 2y = -8$$

'Solve" this system graphically, then check your solution:

Step 1: Rearrange equations into slope intercept form.

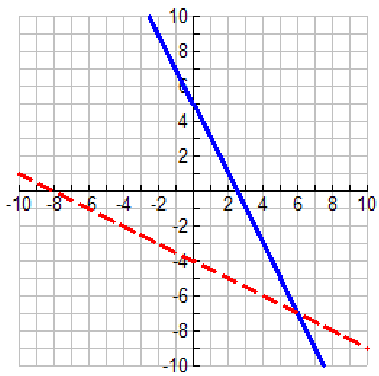
Eq 1 → $2x + y = 5$

$$y = -2x + 5$$

Equation 2 → $x + 2y = -8$

$$2y = -x - 8$$
$$y = \frac{-1}{2}x - 4$$

Step 2: Graph both lines



Step 3: State the point of intersection.

(6, -7)

Step 4: Check your solution in BOTH equations.

$$\begin{aligned} 2x + y &= 5 \\ 2(6) - 7 &= 5 \\ 12 - 7 &= 5 \\ 5 &= 5 \end{aligned}$$

$$\begin{aligned} x + 2y &= -8 \\ 6 + 2(-7) &= -8 \\ 6 - 14 &= -8 \\ -8 &= -8 \end{aligned}$$

Example #2

Without graphing the system, determine if the given point is the solution.

$$\begin{aligned} 3x-2y&=8 \\ 4x+y&=7 \end{aligned} \qquad (2,-1)$$

Substitute the value of *x* with 2 and *y* with -1.

If the point given satisfies both equations (makes them both balanced), then the point given is a solution.

$$\begin{aligned} 3x-2y&=8 \\ 3(2)-2(-1)&=8 \\ 6+2&=8 \\ 8&=8 \end{aligned} \qquad\qquad \begin{aligned} 4x+y&=7 \\ 4(2)+(-1)&=7 \\ 8-1&=7 \\ 7&=7 \end{aligned}$$

Therefore (2, -1) is a solution.

Example #3

Determine the ***number of solutions*** to the following systems of linear equalities:

Step 1 – Write both equations in slope-intercept form.

Step 2 – Use slope and *y*-intercept to determine which of the three cases it satisfies (see page 1)

(a) Equation 1 → $2x+y=3$
Equation 2 → $y-8=-2x$

Equation 1 → $y=-2x+3$ Equation 2 → $y=-2x+8$

Slopes are the same – different *y*-intercepts – lines are parallel so NO solutions.

(b) Equation 1 → $x+y=3$
Equation 2 → $2x+2y=6$

Equation 1 → $y=-x+3$ Equation 2 → $2y=-2x+6$
$y=-x+3$

Same slope and same *y*-intercept – lines are the same so INFINITE solutions

Concept Sheet #4&#5: Solving Linear Systems Algebraically(Substitution & Elimination)

Vocabulary & Key Concepts

Substitution Method

- An **algebraic method** of solving a system of equations that transforms a system of two equations into a single equation in one variable.

- ***Summary***: Isolate one variable in one equation, substitute that expression for the same variable in the other equation and then solve for the variable. Then, replace the value of the one variable into either of the equations to solve for the second variable.

- This method works best when the system has an equation where a variable has a coefficient of 1 – it is best to avoid using fractions **if possible**.

  For example: $x+2y=4$ → x can be isolated without fractions
  $3x-y=7$ → y can be isolated without fractions
  So substitution is a good choice of methods.

Elimination Method

- An **algebraic method** of solving a system of equations that also transform sa system of two equations into a single equation in one variable.

- ***Summary:*** Two equations are added or subtracted in order to 'eliminate' a variable. You can then solve for the remaining variable, and then substitute that value into either equation to solve for the second variable.

- This method is a good choice when there are no coefficients of the variables in the equations equal to 1 where isolating a variable would result in fractions.

  ie) $\begin{array}{l} 2x+3y=4 \\ 3x-4y=8 \end{array}$ → isolating x or y would result in a fraction

  Instead – multiply the first equation by 4 and the second equation by 3 to make the coefficients of y equal, but opposite. By adding the equations, the y would be eliminated and you can solve for the x. Then, plug the value of x back into either equation to solve.

Skills:

Solving Problems using the Substitution Method

Example #1

A system of linear equations is defined by the equations:

$$\begin{aligned} &(1)\quad x+4y=6 \\ &(2)\quad 2x-3y=1 \end{aligned}$$

Solve this system by ***substitution***.

Step 1: *Isolate a variable***.** (x in equation (1))

$$\begin{aligned} (1)\quad x+4y&=6 \\ x&=-4y+6 \end{aligned}$$

Step 2: *Substitute into* (2) *and solve for y.*

$$\begin{aligned} 2(-4y+6)-3y&=1 \\ -8y+12-3y&=1 \\ -11y&=-11 \\ y&=1 \end{aligned}$$

Step 3: *Substitute y into either* (1) or (2) and solve for x.

$$\begin{aligned} x+4y&=6 \\ x+4(1)&=6 \\ x+4&=6 \\ x&=2 \end{aligned}$$

Step 4: *Check your answer in both equations.*

$$\begin{aligned} x+4y&=6 \qquad & 2x-3y&=1 \\ (2)+4(1)&=6 & 2(2)-3(1)&=1 \\ 2+4&=6 & 4-3&=1 \\ 6&=6 & 1&=1 \end{aligned}$$

Example #2

Admission to the 2009 Abbotsford International Airshow cost $80 for a car with two adults and three children. Admission for a car with two adults and a child cost $50. Use algebra to determine the cost for one child and the cost for one adult.

Step 1: *Choose 2 variables and define what each will represent in this situation.*

a = cost per adult
c = cost per child

Step 2*: Write a system of equations using the information in the problem.*

$$\begin{aligned} &(1)\quad 2a+3c=80 \\ &(2)\quad 2a+c=50 \end{aligned}$$

Step 3: *Solve the system of equations algebraically.*

(2) $c = 50 - 2a$ sub back into (1)

$$\begin{aligned} 2a + 3(50 - 2a) &= 80 \\ 2a + 150 - 6a &= 80 \\ -4a &= -70 \\ a &= 17.50 \end{aligned} \qquad \begin{aligned} 2(17.50) + 3c &= 80 \\ 35 + 3c &= 80 \\ 3c &= 45 \\ c &= 15 \end{aligned}$$

Step 4: *Verify your solution.*

$$\begin{aligned} 2a + 3c &= 80 \\ 2(17.50) + 3(15) &= 80 \\ 35 + 45 &= 80 \\ 80 &= 80 \end{aligned} \qquad \begin{aligned} 2a + c &= 50 \\ 2(17.50) + 15 &= 50 \\ 35 + 15 &= 50 \\ 50 &= 50 \end{aligned}$$

Step 5: *Write a sentence to answer the initial problem.*

Therefore, the cost for a child to attend the Airshow is $15 and the cost for an adult is $17.50.

Solving Problems using the Elimination Method

Example #3

A system of linear equations is defined by the equations:

$$3a + b = 5$$
$$9a - b = 15$$

Solve this system by ***elimination***.

Step 1: *Add the equations:*

$$\begin{array}{r} 3a + b = 5 \\ \underline{9a - b = 15} \\ 12a = 20 \end{array}$$

$$a = \frac{20}{12}$$

$$a = \frac{5}{3}$$

Step 2: *Sub back into one of the original equations to solve for b.*

$$3\left(\frac{5}{3}\right)+b=5$$
$$5+b=5$$
$$b=0$$

Step 3: *Check your answer in both equations.*

$$3a+b=5 \qquad 9a-b=15$$
$$3\left(\frac{5}{3}\right)+0=5 \qquad 9\left(\frac{5}{3}\right)-0=15$$
$$\frac{15}{3}+0=5 \qquad \frac{45}{3}-0=15$$
$$5+0=5 \qquad 15-0=15$$
$$5=5 \qquad 15=15$$

Example #4

A system of linear equations is defined by the equations:

$$3x+4y=29$$
$$2x-5y=-19$$

Solve this system by ***elimination***.

$$\begin{array}{l} -2(3x+4y=29) \\ 3(2x-5y=-19) \end{array} \longrightarrow \begin{array}{r} -6x-8y=-58 \\ \underline{6x-15y=-57} \\ -23y=-115 \\ y=5 \end{array}$$

Sub back into original equation:

$$3x+4y=29$$
$$3x+4(5)=29$$
$$3x+20=29$$
$$3x=9$$
$$x=3$$

Check your answer

$$3x+4y=29 \qquad 2x-5y=-19$$
$$3(3)+4(5)=29 \qquad 2(3)-5(5)=-19$$
$$9+20=29 \qquad 6-25=-19$$
$$29=29 \qquad -19=-19$$

Example #5

Judith downloaded two orders of games and songs. The first order consisted of five games and four songs for $26. The second order consisted of three games and two songs for $15. All games cost the same amount, and all songs cost the same amount. Write a system of linear equations. Then, determine the cost of one song and the cost of one game.

Assign variable: x = cost of one game
y = cost of one song

Set up equations:

$$\begin{aligned} 5x + 4y &= 26 \\ 3x + 2y &= 15 \end{aligned}$$

Solve:

$$\begin{aligned} 5x + 4y &= 26 \\ -2(3x + 2y &= 15) \end{aligned} \quad \rightarrow \quad \begin{aligned} 5x + 4y &= 26 \\ -6x - 4y &= -30 \\ \hline -x &= -4 \\ x &= 4 \end{aligned}$$

Substitute back into original:

$$\begin{aligned} 5(4) + 4y &= 26 \\ 20 + 4y &= 26 \\ 4y &= 6 \\ y &= \frac{6}{4} \\ y &= 1.5 \end{aligned}$$

Therefore, the cost of a game is $4 and the cost of a song is $1.50.