## *Math 10 C: Shape & Space*

### *C1 & C2 Measurements & Unit Conversions*

### *Vocabulary & Key Concepts*

***Referent –*** an object used to estimate a measure; for example, a referent for a measure of 1 mm is the thickness of a dime.

***The SI (Metric) System***

The measurement system in Canada is the SI System.

SI (Systeme International d'Unites)
- a measurement system that is based on the base of 10.
- The base unit of measure, for length,  is metres

Using these prefixes we can apply the conversions to grams (mass), litres (volume) and others.

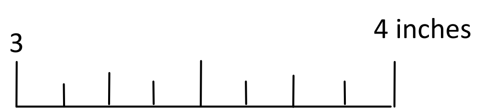
| Unit | Abbreviation | Multiplying Factor |
|---|---|---|
| kilometre | km | 1000 |
| hectometre | hm | 100 |
| decametre | dam | 10 |
| metre | m | 1 |
| decimetre | dm | 0.1 |
| centimetre | cm | 0.01 |
| millimetre | mm | 0.001 |

***The Imperial System***

Most of the world uses SI measurement, although the United States uses the **Imperial Measurement System**, which is based on British units. The imperial system was used in Canada prior to 1976.

Measuring devices using this system often have each unit subdivided by halving and then halving the subdivisions.

**Ex)** Determine the smallest indicated unit on the diagram of an inch ruler below.




***Solution:*** $\frac{1}{8}$ *of an inch*

Conversion within Imperial units is more difficult to do by hand and should be done using ratios and a chart such as this one.

When we move into the Imperial System the use of fractions and decimals become more prevalent.

The abbreviations and conversions for the imperial system are:

| | |
|---|---|
| inch (in. or ″) | |
| foot (ft or ′) | 1 ft = 12 in. |
| yard (yd) | 1 yd = 3 ft or 36 in. |
| mile (mi) | 1 mi = 1760 yd or 5280 ft |

***Converting Between SI and Imperial***

To convert between measurement systems we need to understand the *relationship between units* and *proportions*, much like we did within a measurement system.

The following are some common conversions.

**Exact Conversions**

| | | |
|---|---|---|
| 1 in. = 2.54 cm | 1 ft = 30.48 cm | 1 yd = 0.9144 m |

**Approximate Conversions**

| | | |
|---|---|---|
| 1 mm ≈ 0.0394 in. | 1 cm ≈ 0.3937 in. | 1 m ≈ 1.094 yd |
| 1 m ≈ 3.281 ft | 1 km ≈ 0.6214 mi | 1 mi ≈ 1.609 km |

***Two Methods to Convert Units***

1. **Proportional Reasoning –** set up equivalent ratios and cross multiply to solve for a missing value.

   **For Example:** Convert 96 cm to in.

   $$\frac{96\ cm}{x\ in} = \frac{2.54\,cm}{1\ in}$$

   $$96 = 2.54x$$

   $$x = 37.8\ in$$

2. **Unit Analysis –** Set up a multiplication statement using equivalencies which will cancel out the units you are converting from and leave you with the units you are converting to.

   **For Example:** Convert 96 cm to in.

   $$96\,cm \times \left(\frac{1in}{2.54\,cm}\right)$$

   $$96\,\cancel{cm} \times \left(\frac{1in}{2.54\,\cancel{cm}}\right)$$

   $$x = \frac{96}{2.54}in$$

   $$x = 37.8\ in$$

***Skills***

***Converting within the Metric System***

**Example #1**

Use your formula sheet or the metric line below to convert the following units.

| k | h | da | L<br>m<br>g | d | c | m |
|---|---|---|---|---|---|---|

(a) 30 km = ______________ m

***Solution:*** *To move from k to m on the scale above, you must move 3 spaces to the right, so add 3 zeros onto 30 km (multiply 30 by 1000)*

***30 km = 30 000 m***

(b) 42 307 cm = ____ km

***Solution:*** *To move from cm to km on the scale above, you must move 5 spaces to the left, so move the decimal 5 spaces on the given number*

***42307 cm = 0.42307 km***

**Try:**

(c) 2.7 cm = ______________ mm

(d) 40.7 dam = _______________ km

(e) 5.21 g = _______________ mg

(f) 0.315 kg = ________________ hg

(g) 4.7 hL = ______________ dL

(h) 3085 mL = _______________ L

**Answers:**

**(c)** 27 mm **(d)** 0.407 km **(e)** 5210 mg **(f)** 3.15 hg **(g)** 4700 dL **(h)** 3.085 L

***Converting within the Imperial System***

| | |
|---|---|
| inch (in. or ″) | |
| foot (ft or ′) | 1 ft = 12 in. |
| yard (yd) | 1 yd = 3 ft or 36 in. |
| mile (mi) | 1 mi = 1760 yd or 5280 ft |

**Example #2**

Use the conversions above to convert the following units.

(a) 5 feet = _______________ inches

***Solution:*** $\frac{feet}{inches} \rightarrow \frac{1}{12} = \frac{5}{x}$

$x = 5 \times 12$
$x = 60$ inches

(b) 5.2 pounds = ______________ ounces

***Solution:*** $\frac{pounds}{ounces} \rightarrow \frac{1}{16} = \frac{5.2}{x}$

$x = 16 \times 5.2$
$x = 83.2$ inches

**Try:**

(c) 207 inches = ___________ yards

(d) 12 feet 2 inches = __________ inches

(e) 32560 yards = __________ miles

(f) 6 yards 2 feet = ____________ feet

**<u>Answers</u>**
**(c)** 5.75 yards **(d)** 146 inches **(e)** 18.5 miles **(f)** 20 feet

**Example #3**

Dave knows he is 5 feet 7 inches tall. When he goes to get his driver's license, he must give his height in cm. **Determine** Dave's height to the nearest cm.

***Solution:*** **Step 1:** Convert 5 feet into inches.

$$\frac{feet}{inches} \rightarrow \frac{1}{12} = \frac{5}{x}$$

$$x = 5 \times 12$$
$$x = 60 \text{ inches}$$

**Step 2:** Combine feet measurement and inches measurement.

60 inches + 7 inches = 67 inches

**Step 3:** Convert inches to cm.

$$\frac{inches}{cm} \rightarrow \frac{1}{2.54} = \frac{67}{x}$$

$$x = 2.54 \times 67$$
$$x = 170 \text{ cm}$$

**Try:**

Maggie harvested the potatoes in her garden and found that each potato plant yielded 10 potatoes. Each potato had a mass of 180 g. **Determine** how many kilograms of potatoes Maggie harvested from the 14 plants in her garden?

***Solution:*** *252 kg*

***Converting between the Metric and Imperial System***

| **Relationships between Common Imperial Units and Metric Units** | |
|---|---|
| 1 inch = 2.54 cm | 1 cm = 0.3937 inches |
| 1 mile = 1.609 km | 1 km = 0.6214 miles |
| 1 yard = 0.9144 m | 1 m = 1.0936 yards |
| 1 foot = 0.3048 m | 1 m = 3.2808 feet |

**Example #4**

Use the table above to complete the following metric and imperial conversions:

(a) 114 in = ________________ m

***Proportional Reasoning:***

$$\frac{114\ in}{x\ cm} = \frac{1\ in}{2.54\ cm}$$

$$x = 114 \times 2.54$$

$$x = 289.56\,cm$$

*Convert cm to m -*

$$289.56 \div 100 = 2.90m$$

***Unit Analysis***

$$114\ in \times \left(\frac{2.54\ cm}{1\ in}\right)$$

$$x = 114 \times 2.54$$

$$x = 289.56\,cm$$

*Convert cm to m -*

$$289.56 \div 100 = 2.90m$$

(b) 52 km = ____________mi

$$\frac{52\,km}{x\ mi} = \frac{1\ km}{1.609\,mi}$$

$$x = 52 \times 1.609$$

$$x = 83.67\ mi$$

***C3 & C4: Surface Area and Volume***

***Vocabulary and Key Concepts***

**Surface Area:** Total area of all faces of an object. Common units used: $cm^2, m^2, units^2, inches^2$

***Area*** is the number of square units that will cover a shape. ***Surface area*** is the combined area of all of the surfaces of a 3-dimentional shape.

**Volume:** Amount an object can hold. Common units used: $cm^3, m^3, units^3, inches^3$

**Volume** can be defined as capacity of an object.
(i.e.: how much liquid an object will hold.)

**Note**: Formulas for Surface Area and Volume will be provided on exams and quizzes. However, you should have a good understanding of what the components of the formulas represent.

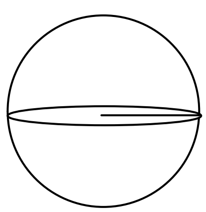
***Skills***

**Example #1**

**Determine** the surface area of each of the following shapes, rounded to the nearest tenth. Write the dimensions on the nets for the cube, cylinder, and rectangular-based prism.

(a) Sphere

$SA = 4\pi r^2$



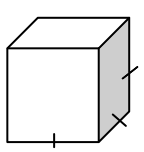
$r$ = 4 cm

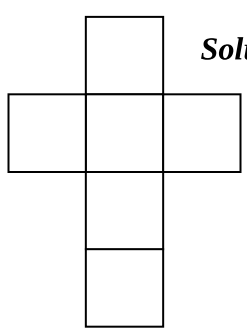
***Solution:***

$SA = 4\pi(4)^2$
$SA = 201.1cm^2$

(b) Cube

$SA = 6s^2$



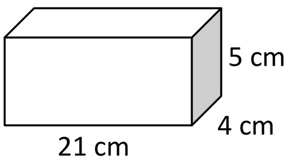



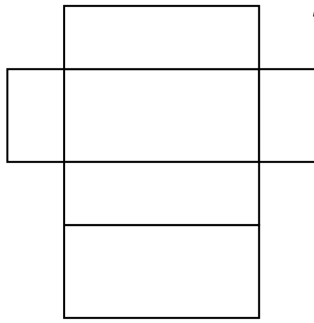
***Solution:***

$SA = 6(12)^2$
$SA = 864.0\,m^2$

(c) Rectangular Prism

$SA = 2lw + 2lh + 2wh$



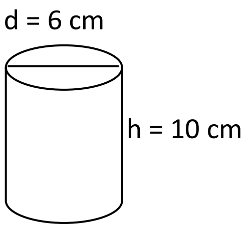



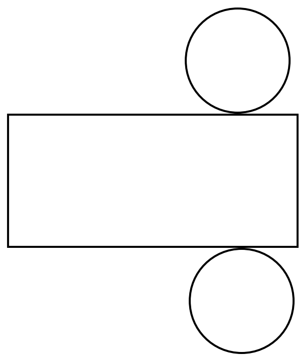
***Solution:*** $SA = 2(21)(4) + 2(21)(5) + 2(4)(5)$

$SA = 168 + 210 + 40$

$SA = 418.0\ cm^2$

(d) Cylinder



$SA = 2\pi r^2 + 2\pi rh$

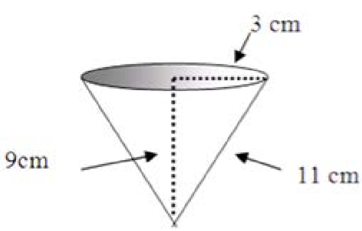
***Solution:*** $SA = 2\pi(3)^2 + 2\pi(3)(10)$

$SA = 56.52 + 188.4$

$SA = 245.0\, cm^2$

(e) Right Cone:

$SA = \pi rs + \pi r^2$

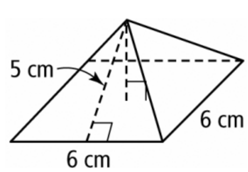



***Solution:*** $SA = \pi(3)(11) + \pi(3)^2$

$SA = 131.9 cm^2$

Right Pyramid:

$SA = lw + 2\left(\frac{1}{2}al\right) + 2\left(\frac{1}{2}sw\right)$




***Solution:*** $SA = (6)(6) + 4\left(\frac{1}{2}(6)(5)\right)$

$SA = 36 + 60$

$SA = 96 cm^2$

**Try:**

The circumference of a ball hockey ball is 24 cm.

(a) Using the circumference formula, **determine** the radius of the ball hockey ball, rounded to the nearest hundredth.

***Answer:*** $3.82\,cm$

(b) **Determine** the surface area of the ball hockey ball, rounded to the nearest tenth.

***Answer:*** $183.4\ cm^2$

(c) If the plastic coating on the outside of the ball costs $0.01 per $cm^2$, **determine** what it costs for the plastic coating.

***Answer:*** $1.83

***Working Backwards***

Sometimes, we know the surface area of a sphere but we need to solve for the radius. Manipulate the variables of the surface area formula to **solve** for the radius, r.

**Example #2**

(a) A sphere has a surface area of 100 $cm^2$. **Determine** the radius of the sphere, to the nearest hundredth by substituting the values into the formula and solving for the radius, r.

$$SA = 4\pi r^2$$
$$100 = 4\pi r^2$$
$$\frac{100}{4\pi} = r^2$$
$$r^2 = 7.96$$
$$r = 2.82cm$$

**Try:**

(b) A sphere has a surface area of 50 $cm^2$. **Determine** the diameter of the sphere, to the nearest hundredth.

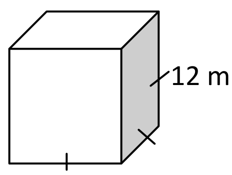
***Answer:*** 3.99 cm

***Volume***

**Example #**

**Determine** the volume of each of the following shapes, rounded to the nearest tenth. Identify the name of the shape first.

(a) Cube

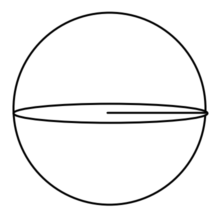



$V = s^3$

***Solution:*** $V = 12^3$

$V = 1728.0\ m^3$

(b) Sphere



$r$ = 4 cm

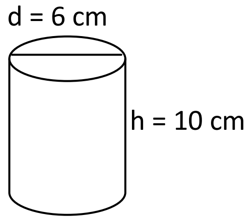
$V = \frac{4}{3}\pi r^3$

***Solution:*** $V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi (4)^3$

$V = 267.9\ cm^3$

(c) Cylinder

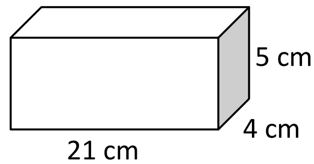



$$V = \pi r^2 h$$

$$r = \frac{1}{2} diameter \rightarrow r = 3cm$$

$$V = \pi(3)^2(10)$$
$$V = 282.7\,cm^3$$

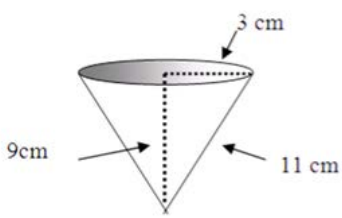
(d) Rectangular Prism




$$V = lwh$$

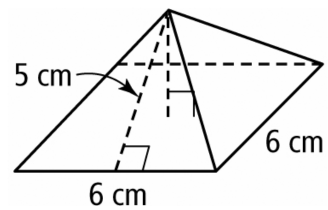
$$V = (21)(4)(5)$$
$$V = 420.0cm^3$$

(e) Right Cone:




$$V = \frac{1}{3}\pi r^2 h$$
$$V = \frac{1}{3}\pi(3)^2(9)$$
$$V = 84.8cm^3$$

(f) Right Pyramid




$$V = \frac{1}{3}lwh$$
$$V = \frac{1}{3}(6)(6)(5)$$
$$V = 60cm^3$$

**Try**

**Determine** the volume of a sphere with a diameter of 9 cm, rounded to the nearest tenth.

$$V = \frac{4}{3}\pi r^3$$

***Answer:*** $381.7\ cm^3$

***Working Backwards with Volume***

Sometimes, we know the volume of a sphere but we need to solve for the radius. Manipulate the variables of the volume formula to solve for the radius, r.

**Example #4**

A sphere has a volume of 100 $cm^3$. **Determine** the radius of the sphere, to the nearest hundredth by substituting the values into the formula and solving for the radius, r. Recall: $V = \frac{4}{3}\pi r^3$

***Solution:*** $100 = \frac{4}{3}\pi r^3$

$$3 \times 100 = 4\pi r^3$$

$$\frac{300}{4\pi} = r^3$$

$$r^3 = 23.8854$$

$$r = \sqrt[3]{23.8854}$$

$$r = 2.88\ cm$$

**Example #4**

A sphere has a volume of 50 $cm^3$. **Determine** the diameter of the sphere, to the nearest hundredth by substituting the values into the formula and solving for the radius, r. Then, **determine** the diameter of the sphere.

$$V = \frac{4}{3}\pi r^3$$

***Answer**: 4.57 cm*