**Chapter 7 Absolute Value and Reciprocal Functions Concept Review**

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| **Key Ideas** | **Description or Example** |
| **Absolute value** | Represents the distance from zero on a number line, regardless of direction. Absolute value is written with a vertical bar around a number or expression. It represents a positive value.  Example: |24| = 24  |-2| = 12  The absolute value of a positive number is positive, The absolute value of a negative number is positive, and the absolute value of zero is zero.    Piecewise Definition of an absolute value function |
| **Graphing an Absolute Value Function** | To graph the absolute value of a linear function:   * Method 1: Create a table of values, then graph the function using the points. Note: There are two pieces and all points are on or above the x-axis. * Method 2: Graph the related linear function y = 2x + 3. Reflect, in the x-axis, the part of the graph that is below the x-axis. (Negative y-values become positive.)   The domain is all real numbers.  The range is .  To graph the absolute value of a quadratic function:   * Method 1: Create a table of values, and then graph the function using the points. Note there are three pieces and all points are on or above the x-axis. * Method 2: Graph the related quadratic function   y = x2 – 4. Then reflect in the x-axis the part of the graph that is below the x-axis.  The domain is all real numbers.  The range is . |
| **Writing Absolute Value as a Piecewise Function**  1. Determine the x-intercepts by setting the expression within the absolute value equal to zero.  2. Use slope (linear function) or direction of opening (quadratic function) to determine which parts of the graph are above or below the x-axis.  3. Keep the parts that are positive (above x-axis) and indicate the domain.  4. Reflect the negative parts in the x-axis, multiply the expression by -1 for this part and indicate the domain. | Be careful when assigning domain, it changes depending on which piece of the graph was below the x-axis.  Note the linear expression would have a negative slope, examine how this changes the domain pieces.  Linear Expressions  Quadratic Expressions |
| **Analyzing Absolute Value Functions Graphically** | To analyze an absolute value graphically:   * first, graph the function. * then, identify the characteristics of the graph, such as x-intercept, y-intercept, minimum values, domain and range   The domain of an absolute value function, y = |f(x)|, is the same as the domain of y = f(x).  The range of the absolute value function will be greater or equal to zero. |
| **Solving an Absolute Value Equation Graphically** | An absolute value equation includes the absolute value of an expression involving a variable. To solve an graphically:   * Graph the left side and the right side of the equation on the same set of axes. * The point(s) of intersection are the solutions. |
| **Solving an Absolute Value Equation Algebraically**  **Determine the zero of the function inside the abs.**  **Use sign analysis to determine which parts of the domain are positive or negative.** | To solve algebraically consider the two cases:   1. Case 1- the expression inside the absolute value symbol is greater than or equal to zero. 2. Case 2- the expression in the absolute value symbol is less than zero.  * The roots in each case are the solutions. * There may be extraneous roots that need to be identified and rejected. * Verify the solution by substituting into the original equation.   There may be no solution, one solution or two solutions if the absolute value expression is a line.  There may be no solution, one, two or three solutions if the absolute value expression is quadratic. |
| **Solving Linear Abs Equation** | Determine zero of the abs function. Determine domain pieces.    Case 1: positive y values Case 2: negative y values      Verify each solution in the original absolute value equation.  |*x* + 3| = - 2 does not have any solution, absolute value must be positive. |
| **Solving Quadratic Abs Equations** | Determine the zeros of the abs function.        continued on next page  Case 1: positive y values Case 2: negative y values      Solutions are in the domain. Neither solution is in domain.  These solutions are extraneous. |
| **Reciprocal Functions**  **A reciprocal function has the form y =**  **where *f*(*x*) is a polynomial and**  ***f*(*x*) ≠ 0.** | For any function f(x), the reciprocal function is . The reciprocal of y = x is y = 1/x. |
| **To Graph a reciprocal Function:** | * Plot invariant points. Invariant points are where the y values are 1 or -1 * x-intercepts become vertical asymptotes. * The x-axis is a horizontal asymptote * Take the reciprocal of the y values of the original function to plot the reciprocal of the function. |
| **Restrictions on the denominator of the reciprocal function.** | The reciprocal function is not defined when the denominator is 0. These non-permissible values relate to the asymptotes of the graph of the reciprocal function.  The non-permissible values for a reciprocal function (position of asymptotes) also come from the x-intercepts of the original graph. |
| **Linear Reciprocal Graphs** | Quadratic Reciprocal Graphs |

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| **Vocabulary** | **Definition** |
| **Absolute Value** | Distance from zero on a number line.  Piecewise definition |
| **Absolute Value Function** | A function that involves the absolute value of a variable. |
| **Piecewise Function** | A function composed of two or more separate functions or pieces, each with its own specific domain, that combine to define the overall function. |
| **Invariant Point** | A point that remains unchanged when a transformation is applied to it. |
| **Absolute Value Equation** | An equation that includes the absolute value of an expression involving a variable. |
| **Asymptote** | An imaginary line whose distance from a given curve approaches zero. |
| **Vertical Asymptote** | For reciprocal functions, vertical asymptotes occur at the non-permissible values of the function, the x-intercepts of the original function graph. |
| **Horizontal Asymptote** | For our reciprocal functions, there will always be a horizontal asymptote at y = 0. |

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| **Common Errors** | **Description** |
| No vertical bars around the value. | A small error that sometimes seems to happen are students forgetting to put vertical bars around the value, which does not get the absolute value of whatever the value is. |
| Not eliminating extraneous solutions when it comes to Absolute Value equations. | When verifying a solution, if the solution does not satisfy the equation, the solution is considered extraneous. |
| Not plotting the invariant points in the proper areas. | An error that may occur is plotting invariant points, on the wrong co-ordinate of the line. |
| Not plotting the Vertical or Horizontal asymptote in the proper place. | If the vertical asymptote is not plotted in the proper area, the entire graph is wrong. |
| Not properly graphing Absolute Value functions. | Some students graph absolute value functions as if it was a linear function and forget that it is an absolute value graph; no part of the graph will be in the negative portion of the co-ordinate plane. |