Math 20-2

Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Date: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Unit Six



Logic and Reasoning

**1.1 Inductive Reasoning**

***Learning Outcome:***

*Learn to use reasoning to make predictions.*

***Investigation:***

Complete the following investigation with a partner:

Frank has been patterning with equilateral triangles.

Figure 3

Figure 2

Figure 1

Frank thinks that Figure 10 in this pattern will have 100 triangles, and all these triangle will be congruent to the triangle in figure 1.

How did Frank arrive at this conjecture? (Discuss with a partner what conjecture means)

1. Organize the information about the pattern the table below:

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Figure | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Number of Triangles | 1 | 4 |  |  |  |  |  |  |

1. What do you notice about the data in the table?
2. Extend the pattern for two more figures.
3. What numeric pattern do you see in the table?
4. Is Frank’s conjecture reasonable? Explain.
5. How did Frank use inductive reasoning to develop his conjecture?
6. Is there a different conjecture you could make based upon the pattern you see? Explain.

***Vocabulary:***

Conjecture: A \_\_\_\_\_\_\_\_\_\_\_\_\_\_ expression that is based on available evidence but is not yet \_\_\_\_\_\_\_\_\_\_\_\_.

Inductive reasoning: Draw a general \_\_\_\_\_\_\_\_\_\_\_\_\_ by observing \_\_\_\_\_\_\_\_\_\_\_and identifying properties in specific examples.

**Examples:**

1. What conjecture could you make for the following situations:
	1. Eli has lunch at the same deli every day for a week. Each day, the “Special of the Day” has included fries. Based on this, what can Eli conclude about his next visit to the Deli?
	2. Justin arrives at his first block class at 8:10 am. Every day for the past three days, his teacher has arrived between 8:12 and 8:14 am and the class starts at 8:15. Therefore we can conclude that tomorrow his block one teacher…….
2. With a partner, use inductive reasoning to make a conjecture about the following situations:
	1. Suppose you add two odd numbers; note the result. Repeat the experiment several times. What conclusion can you make?
	2. Multiply an even number by an odd number and note the result. Repeat the experiment several times. What conjecture can you make?
	3. Construct several scalene triangles. Measure each of the angles and each of the sides. Note the relationship between the smallest angle and the measure of the smallest side. What conjecture can you make?
	4. Construct several pairs of intersecting lines. Measure the pairs of vertically opposite angles. What conjecture can you make?

Does a conjecture prove a theory?

**Assignment:** Textbook Page 12 #1-5, 7-13, 15, 16, 19, 20

**1.3 Using Reasoning to Find a Counterexample to a Conjecture**

***Learning Outcome:***

*Learn to invalidate a conjecture by finding a contradiction.*

***Investigation:***What is a counterexample?

1. With a partner, find a counterexample for each of the following situations:
	1. If you live in a country bordering the United States, then you live in Canada.
	2. If a quadrilateral has four right angles, then it is a square.
	3. Odd whole numbers less than 10 are prime.
	4. All birds can fly
	5. All cats are born with a tail.

***Examples:***

1. Consider the conjecture:
The sum of two prime numbers is an even number.
2. Give two examples for which the conjecture is true.
3. Give one counterexample that shows the conjecture is false.
4. Matt found a interesting numeric pattern:

1 x 8 + 1 = 9

12 x 8 + 2 = 98

123 x 8 + 3 = 987

1234 x 8 + 4 = 9876

Matt thinks that this pattern will continue.

Search for a counterexample to Matt’s conjecture.

|  |  |  |
| --- | --- | --- |
|  | A | B |
| 1 | 1 x 8 + 1 | 9 |
| 2 | 12 x 8 + 2 | 98 |
| 3 | 123 x 8 + 3 | 987 |
| 4 | 1234 x 8 + 4 | 9876 |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |
| 9 |  |  |
| ***10*** |  |  |
|  |  |  |

1. Give two examples that support each conjecture, and then one counterexample that shows the conjecture is false.
2. The sum of two perfect squares is an even number.
3. The square root of a number is smaller than the number.

**Assignment:** Textbook Page 22 #1-6, 8, 10-12, 14, 17-19

**1.4 Deductive Reasoning**

***Learning Outcome:***

*Learn to prove mathematical statements using a logical argument.*

What are the limitations of inductive reasoning?

How do you think deductive reasoning will differ?

***Vocabulary:***

Deductive Reasoning: Drawing a specific conclusion through \_\_\_\_\_\_\_\_\_\_\_\_ reasoning by starting with \_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_ that are known to be \_\_\_\_\_\_\_\_\_\_\_.

***Examples:***

* 1. Sam transferred to a new high school. A student must attend school for at least six months before being eligible to play on a school team. Therefore, before he can play on a school team…
	2. Between the hours of noon and 2:00 pm., a bank offers to give you $5 if no one serves you within 5 minutes. Sarah arrived at 1:30 pm, and no one helped her until 1:42 pm. Therefore, …
	3. Paulette lives in Medicine Hat. Medicine Hat is in Alberta. Therefore, …

With a partner, create an example of deductive reasoning.

***Using Deductive Reasoning***

Opposite Angles Theorem:

When two lines intersect the opposite angles are equal.

1

4

3

2

$$∠1= ∠4$$

$$∠2= ∠3$$

We can deductively prove that this is the case:

Similarly, we can use deductive reasoning to show that ∠1 = ∠ 4:

Mike approached his friend Jack saying that he had a mathematical magic trick. He handed Jack a piece of paper and a pencil and had him complete the following steps:

* Select a positive number
* Triple the number
* Subtract two
* Add eight
* Divide by three
* Subtract the original number

Mike said the answer will always be 2.

1. Use inductive reasoning to test the trick to see if Mike’s magic trick will work:
2. Use deductive reasoning to show that the answer will always be two regardless of the original number.
* Select a positive number
* Triple the number
* Subtract two
* Add eight
* Divide by three
* Subtract the original number

By using a variable, x, to represent the chosen number, we have proven deductively that this algorithm will produce an answer of two regardless of the number chosen.

Class discussion:

What was the learning outcome for today’s class?

State the difference between inductive and deductive reasoning.

**Assignment:** Textbook Page 31 #1-5, 7-10, 16, 17

**1.5 Proofs That Are Not Valid**

***Learning Outcome:***

*Learn to identify errors in proofs.*

***Investigation:***

Given the following argument, can you identify any errors or faulty logic?

Professional athletes do not compete in more than one sport. Bo Jackson played football for the Oakland Raiders. Therefore, Bo Jackson has not played in any other professional sports.

***Vocabulary:***

Invalid Proof: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Example:**

Jane claims she has proved that -5 = 5.

Jane’s proof:

I assumed that -5 = 5

Then I squared both sides: (-5)² = 5²

I got a true statement: 25 = 25

This means that my assumption, -5 = 5, must be correct.

Where is the error in Jane’s proof?

Circular Reasoning:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Example:**

Wayne is trying to prove the following number trick:

Choose any number. Add 3. Double it. Add 4. Divide by 2. Take away the number you started with.

Each time Wayne tries the trick, he ends up with 5. His proof, however, does not give the same result.

Wayne’s proof:

Choose any number: n

Add 3: n + 3

Double it: 2n + 6

Add 4: 2n + 10

Divide by 2: 2n + 5

Take away the number you started with: n + 5

Try the number trick yourself. Does it work?

Where is the error in Wayne’s proof?

**Assignment:** Textbook Page 42 #1-3, 5-8, 10

**1.6 Reasoning to Solve Problems**

***Learning Outcome:***

*Learn to solve problems using inductive or deductive reasoning.*

***Investigation:***

With a partner, complete the following investigation:

Emma was given this math trick:

* Choose a number
* Multiply by 6
* Add 4
* Divide by 2
* Subtract 2

Emma was asked to use inductive reasoning to make a conjecture about the relationship between the starting and ending numbers, and then use deductive reasoning to prove that her conjecture is always true. Here is her response to the problem:

**Inductive Reasoning:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **#** | **x 6** | **+4** | **÷2** | **-2** |
| **5** | 30 | 34 | 17 | 15 |
| **-3** | -18 | -14 | -7 | -9 |
| **0** | 0 | 4 | 2 | 0 |
| **24** | 144 | 148 | 74 | 72 |

What conjecture do you think Emma made?

**Deductive Reasoning:**

I chose *d*.

Then I multiplied, added, divided, and subtracted to get an expression.

$$\left(\frac{6d+4}{2}\right)-2$$

It simplified to 3*d*.

1. Explain why Emma might have chosen the values she did?
2. What details are missing from the deductive reasoning Emma used to arrive at the expression 3*d*?
3. How does it help to understand the mathematics when both symbols and words are used in an explanation?

***Examples:***

1. The members of a recently selected basketball team met each other at their first team meeting. Each person shook the hand of every other person. The team had 10 players and 2 coaches. How many handshakes were exchanged?

Is it easier to use deductive or inductive reasoning?

1. Sue signed up for games at her school’s fun night. Seven other people were assigned to her group, making up four pairs of partners. The other members of her group were Dave, Angie, Josh, Tanya, Joy, Stu, and Linus. When the games started, Dave and his partner were to the left of Stu. Across from Dave was Sue, who was to the right of Josh. Dave’s brother’s partner, Tanya, was across from Stu. Joy was not on Stu’s right. Name the four pairs of partners.

Was inductive or deductive reasoning used?

**Assignment:** Textbook Page 48 #1-9, 15, 17, 19

**1.7 Analyzing Puzzles and Games**

***Learning Outcome:***

*Learn to determine, explain, and verify a reasoning strategy to solve a puzzle or to win a game.*

***Investigation A:***

Discuss the following scenario with a partner:

Three students are playing a game. Two of the students flip a coin, and the third student records their scores. Student 1 gets a point if the result is two heads, student 2 gets a point if the result is two tails, and student 3 gets a point if the result is a head and a tail. The first student to get 10 points wins. Explain whether you would prefer to be student 1, student 2 or student 3.

Did your reasoning involve inductive or deductive reasoning?

***Investigation B:***

With a partner, try the game below, then analyze the game to see if there can be any strategy to use.

Toothpick Game: (2 players)

Place a pile of 20 toothpicks on a desk. On each turn a player can take one or two toothpicks from the pile. The player to remove the last toothpick is the winner. Flip a coin to determine the starting player. Is there a strategy you can use to ensure you win the game?

Did your reasoning for strategy involve inductive or deductive reasoning?

**Assignment:** Textbook Page 55 #4-9, 11, 12, 14