Math 20-2

Unit Three

**Math is**



Radicals

Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Date: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**4.1 Mixed and Entire Radicals**

***Learning Outcome:***

*Learn to compare and express numerical radicals in equivalent forms.*

***Investigation:***

What is a radical? Give an example below.

Can you provide an example of a mixed radical and an entire radical?

If we want to change an entire radical to a mixed radical, what steps would we need to follow?

With a partner, come up with a process for changing $\sqrt{52}$ to a mixed radical:

If we wanted to change$\sqrt[3]{54}$ to a mixed radical, would our process outlined above still work? What changes would we have to make to our strategy?

***Examples:***

1. Express each entire radical as a mixed radical in simplest form.
	1. 
	2. $\sqrt[3]{144}$
	3. $\sqrt{288}$
	4. $\sqrt[3]{432}$

If we wanted to go in the opposite direction, converting from a mixed radical to an entire radical, would our strategy need to change?

With a partner, decide on a strategy that would convert $4\sqrt{6}$ to an entire radical:

1. Write each mixed radical as an entire radical:
	1. 7$\sqrt{3}$
	2. 2$\sqrt[3]{4}$
	3. 2$\sqrt[5]{3}$
	4. $-5\sqrt{2}$
	5. $-2\sqrt[3]{5}$

Why did we write the negative sign under the radical sign in *e* but not in *d*.

***Key Ideas:***

A radical is in simplest form when the exponent of the radicand is less than the index of the radical. For example, $12\sqrt{3}$ and $13\sqrt[3]{4}$ are in simplest form, while $12\sqrt{4} $is not.

A square has a principal square root, which is positive, and a secondary square root, which is negative. For example, the principal square root of 16 is $\sqrt{16}$ or 4, and the secondary square root of 16 is -$\sqrt{16}$ or -4. The radical of a square root may be negative, but the radicand of a square root must be positive.

If you express an answer as a radical, the answer will be exact. If you write a radical in decimal form, the answer will be an approximation, except when the radicand is a perfect square. For example, $\sqrt{12}$ expressed as $2\sqrt{3}$ remains an exact value, while $\sqrt{12}$ expressed as 3.464… is an approximation. Both $\sqrt{9}$ and 3 are exact values.

**Assignment:** Textbook Page 182 #4, 5, 7, 9-12, 18, 20

**4.2 Adding and Subtracting Radicals**

***Learning Outcome:***

*Learn to select a strategy to add two radicals.*

***Investigation:***

What is a like term? Can you provide an example of one?

When looking for like radicals, what do we look for?

Given the following radicals, which ones are like radicals?

$$5\sqrt{7}, 2\sqrt{5}, 2\sqrt{3}, -\sqrt{7}$$

Group like radicals in this second group:

$\frac{2}{3}\sqrt[3]{5x^{2}}$, $\sqrt[4]{5a}$, $\sqrt[5]{5a}, \sqrt[3]{5x^{2}}$

What conclusion can we make?

Simplify:



What conclusion can be made about adding/subtracting radicals?

***Examples:***

1. Given the following triangle:

$$\sqrt{50} cm$$

$$10\sqrt{45} cm$$

P

S

Q

$$10\sqrt{8} cm$$

$$29\sqrt{5} cm$$

R

 Determine the difference in length between each pair of sides:

1. PS and SR

$$\sqrt{50}-10\sqrt{8}$$

1. RQ and PQ

$$29\sqrt{5}-10\sqrt{45}$$

1. Simplify radicals and combine like terms:
2. $2\sqrt{7}+13\sqrt{7}$
3. $\sqrt{24}-\sqrt{6}$
4. $2\sqrt{27}-4\sqrt{3}-\sqrt{12}$

**Assignment:** Textbook Page 188 #5-7, 9, 11, 14-16

**4.3 Multiplying and Dividing Radicals**

***Learning Outcome:***

*Learn to multiply and divide numerical radicals.*

***Investigation A:***

Given the following question:

$$\left(2\sqrt{7}\right)\left(4\sqrt{5}\right)$$

Now given the following answer to the question:

$$\left(8\sqrt{35}\right)$$

How do you think we got the solution? Provide your steps. Is there more than one way to approach the question?

***Examples:***

* 1. Express in simplest form:
1. $\sqrt{7}×\sqrt{4}$
2. $2\sqrt{3}\left(\sqrt{12}-\sqrt{7}\right)$
3. $\left(5\sqrt{3}+2\sqrt{6}\right)^{2}$

***Key Ideas:***

The product of two square roots is equal to the square root of the product.

$$\sqrt{3}∙\sqrt{2}=\sqrt{3∙2}=\sqrt{6}$$

The product of two mixed radicals is equal to the product of the rational numbers times the product of the radicals.

$$3\sqrt{2}∙5\sqrt{7}=15\sqrt{14}$$

***Investigation B:***

Given the following that needs to be simplified:

$$\frac{4\sqrt{6}}{2\sqrt{3}}$$

The answer for the given problem is:

$$2\sqrt{2}$$

What steps do we need to take in order to get the solution given?

When dividing radicals, divide the coefficients and then divide the radicands. You can only divide radicals that have the same index.

***Examples:***

1. Express in simplest form:
2. $\frac{6\sqrt{48}}{3\sqrt{6}}$

There are other ways to solve radicals involving division.

Rationalizing Denominators:

To simplify an expression that has a radical in the denominator, you need to rationalize the denominator.

For an expression with a monomial square-root denominator, multiply the denominator and numerator by the radical term from the denominator.

$$\frac{5}{2\sqrt{3}}=\frac{5}{2\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)$$

$$=\frac{5\sqrt{3}}{2\sqrt{3}\left(\sqrt{3}\right)}=\frac{5\sqrt{3}}{6}$$

Is the product equivalent to the original expression? How can we check?

***Example:***

1. Solve $\frac{6\sqrt{48}}{3\sqrt{6}}$ by rationalizing the denominator:
2. Simplify $\frac{4\sqrt{12}-10\sqrt{6}}{2\sqrt{3}}$

***Key Ideas:***

The quotient of two square roots is equal to the square root of the quotient:

$$\frac{\sqrt{6}}{\sqrt{2}}=\sqrt{3}$$

The quotient of two mixed radicals is equal to the product of the quotient of the coefficients and the quotient of the radicals:

$$\frac{15\sqrt{14}}{5\sqrt{7}}=3\sqrt{2}$$

**Assignment:** Textbook Page 198 #4, 5, 8, 11, 13, 14, 16, 19

**4.4 Simplifying Algebraic Expressions Involving Radicals**

***Learning Outcomes:***

*Learn to simplify radical expressions that contain variable radicands.*

***Investigation:***

Given the following radicals:

* determine if there are any numbers the variable can’t be
* explain why
* simplify the expression
* state the restriction
1. $\sqrt{x}$

1. $\sqrt{x^{2}}$
2. $\sqrt{x³}$
3. $\sqrt{x^{4}}$

***Examples:***

1. State any restrictions on the variable, then simplify each expression:
2. $\sqrt{x}+5\sqrt{x}$
3. $4\sqrt{18x^{3}}$
4. $\sqrt{x-5}$
5. $\left(5\sqrt{6x^{2}}\right)\left(-2x\sqrt{2x}\right)$
6. $\left(2\sqrt{x}+3\right)\left(5-3\sqrt{x}\right)$
7. $\frac{15\sqrt{x^{3}}}{-3\sqrt{x^{2}}}$

**Assignment:** Textbook Page 211 #4, 6, 8-12

**4.6 Solving Radical Equations**

***Learning Outcome:***

*Learn to solve and verify radical equations that contain a single radical.*

***Investigation:***

With a partner, solve and verify the following radical equation:

$$\sqrt{3x}=6$$

$$x\geq 0$$

***Vocabulary:***

Extraneous Root: A root that does not satisfy the initial conditions that were introduced while solving an equation. Root is another word for solution.

***Examples:***

1) Solve and verify the following equations and state the restrictions:

* 1. $\sqrt{x+2}=-3$
1. $\sqrt{x-1}+3=4$
2. $\sqrt[3]{2x}=4$
3. The forward and backward motion of a swing can be modeled using the formula:

$$T=2π\sqrt{\frac{L}{9.8}}$$

*T* represents the time in seconds for a swing to return to its original position and *L* represents the length of the chain supporting the swing in metres. If a person swinging took 2.5s to return to its original position, how long is the chain to the nearest centimeter.

Verify the solution and state the restrictions:

**Assignment:** Textbook Page 222 #4, 6-8, 11, 13, 15