# Math 20-2: U6L3 Teacher Notes
# Factored Form of a Quadratic Function

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## Key Math Learnings:
**By the end of this lesson, you will learn the following concepts:**

- determine, with or without technology, the coordinates of the vertex of the graph of a quadratic function.
- determine the equation of the axis of symmetry of the graph of a quadratic function, given the x-intercepts of the graph.
- determine the domain and range of a quadratic function.
- sketch the graph of a quadratic function.
- solve problems that involves the characteristics of a quadratic function.
- determine, with or without technology, the intercepts of the graph of a quadratic function.
- express a quadratic equation in factored form, given the zeros of the corresponding quadratic function or the x-intercepts of the graph of the function.

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## Factored Form of a Quadratic

Another form that a Quadratic function can be written in is Factored Form.

$$y = a(x - r)(x - s)$$

Before we can continue on to understand how factored form is related to the graph of a function, lets review factoring.  Watch the following Brightstorm videos to help you refresh your memory.

brightstorm Greatest Common Factor

brightstorm Difference of Perfect Squares

brightstorm Factoring Trinomials, a = 1

brightstorm Factoring Trinomials, a is not equal to 1

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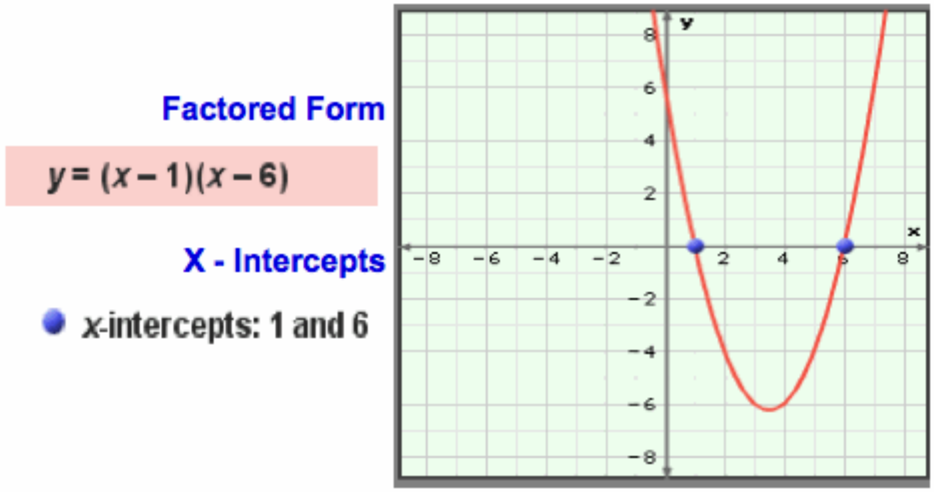
## What is a Zero?

When a quadratic function is written in factored form, each factor can be used to determine a zero of the function by setting each factor equal to zero and solving.




Please note from the above picture, that the zeros of a quadratic function correspond to the *x*-intercepts of the parabola that is defined by the function.

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## The Number of X-Intercepts

### 1. Two X-Intercepts

If a parabola has one or two *x*-intercepts, the equation of the parabola can be written in factored form using the *x*-intercept(s) and the coordinates of one other point on the parabola.

### 2. One X-Intercepts

If a quadratic function has only one *x*-intercept, the factored form can be written as follows:

$f(x) = a(x - r)(x - r)$
$f(x) = a(x - r)^2$

### 3. No X-Intercepts

Quadratic functions without any zeros cannot be written in factored form.

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**Example**

For the quadratic function below

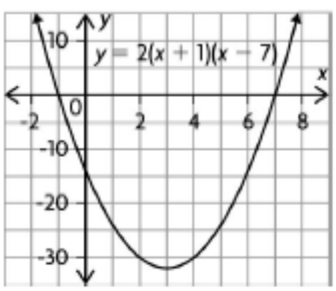
$h(x) = 2(x + 1)(x - 7)$

**a)** determine the *x*-intercepts of the graph
**b)** determine the *y*-intercept of the graph
**c)** determine the equation of the axis of symmetry
**d)** determine the coordinates of the vertex
**e)** sketch the graph

***Solution:***
a. The x-intercepts are –1 and 7.
b.
f(0) = 2(1)(–7)
f(0) = –14
The y-intercept is –14.

c. $\frac{-1+7}{2} = 3$, so the equation of the axis of symmetry is x = 3

d.
$f(3) = 2(3 + 1)(3 - 7)$
$f(3) = (8)(-4)$
$f(3) = -32$
The vertex is (3, –32)
e) Here is the diagram.

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**Example**
For the quadratic function below

$f(x) = (x - 1)(x + 1)$

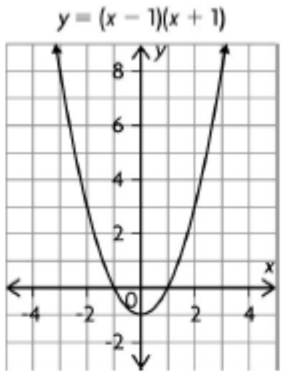
**a)** determine the *x*-intercepts of the graph
**b)** determine the *y*-intercept of the graph
**c)** determine the equation of the axis of symmetry
**d)** determine the coordinates of the vertex
**e)** sketch the graph

*Solution:*

a) The *x*-intercepts are 1 and –1.
b) The *y*-intercept is (0 – 1)(0 + 1) = –1.
c)

$$\frac{-1+1}{2} = 0$$

The equation of the axis of symmetry is *x* = 0.
*d)* *f*(0) = –1
The vertex is (0, –1).
e)

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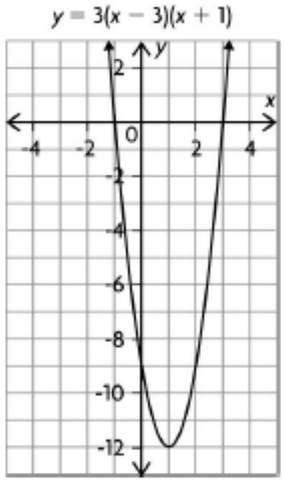
**Example**

Sketch the graph of $y = a(x-3)(x+1)$

for *a* = 3. Describe how the graph would be different from your sketch if the value of *a* were 2, 1, 0, -1, -2, and -3.

***Solution:***
For *a* = 3, *x*-intercepts: 3 and –1
*y*-intercept: –9
Equation of the axis of symmetry: *x* = 1.
Vertex: (1, –12)
Opens upward.

x 


For *a* = 1 or 2, the vertex would be closer to the *x*-axis and the *x*-intercepts would be the same, so the graph would be compressed vertically.
For *a* = 0, the graph would be a straight line, the *x*-axis (*y* = 0).
For *a* = –3, the graph would be a reflection of the graph for *a* = 3, and would open downward.
For *a* = –1 or –2, the graph would be a reflection of the graph for *a* = 1 or 2, so it would be more compressed that the graph for *a* = 3 and open downward.

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**Example**
For the quadratic function below

$$f(x) = x^2 - 8x + 13$$

**a)** use partial factoring to determine two points that are the same distance from the axis of symmetry
b) determine the coordinates of the vertex
**c)** sketch the graph

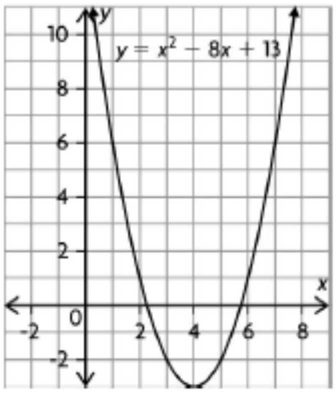
***Solution:***
a)
f(x) = x(x – 8) + 13
f(0) = 13
f(8) = 13
The two points are (0, 13) and (8, 13).

b)

$$\frac{0+8}{2} = 4$$

f(4) = 4(4 – 8) + 13
f(4) = – 3
The vertex is (4, –3).

c)

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**Example**

For the quadratic function below

$$f(x) = -\frac{1}{2}x^2 + 2x - 3$$

**a)** use partial factoring to determine two points that are the same distance from the axis of symmetry

b) determine the coordinates of the vertex

**c)** sketch the graph

***Solution:***

a)

$$f(x) = -\frac{1}{2}x(x-4) - 3$$

f(0) = –3

f(4) = –3

The two points are (0, –3) and (4, –3).

b)

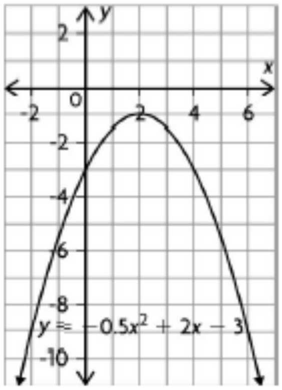
$$\frac{0+4}{2} = 2$$

$$f(2) = -\frac{1}{2}(2)(2-4) - 3$$

$f(2) = -1$

The vertex is (2, –1).

c)

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**Example**
For the quadratic function below
$f(x) = -2x^2 + 10x - 9$

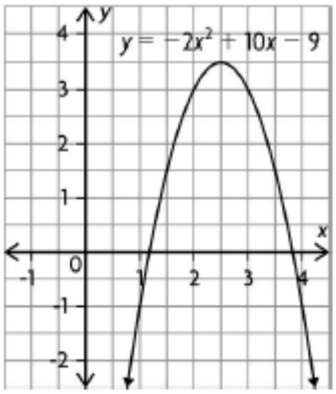
**a)** use partial factoring to determine two points that are the same distance from the axis of symmetry
b) determine the coordinates of the vertex
**c)** sketch the graph

***Solution:***
***a)***
$f(x) = -2x(x - 5) - 9$
$f(0) = -9$
$f(5) = -9$
The two points are (0, –9) and (5, –9).

***b)***
$\frac{0+5}{2} = 2.5$
$f(2.5) = -2(2.5)(2.5 - 5) - 9$
$f(2.5) = 3.5$

The vertex is (2.5, 3.5).

***c)***

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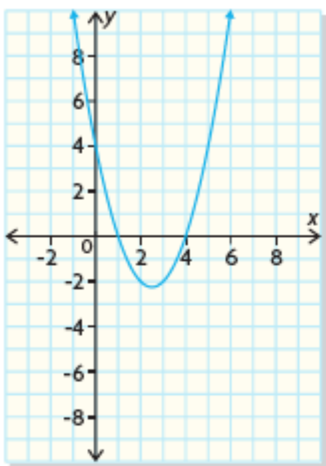
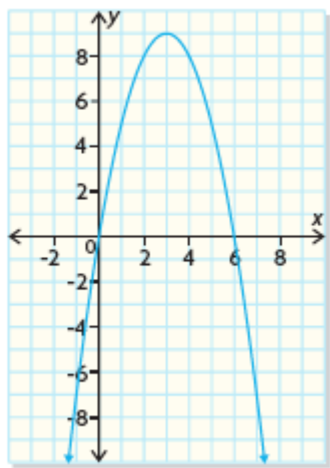
**Example**

Determine the equation of the quadratic function that defines each parabola.




a. b.

***Solution:***

*a.*

The $x$-intercepts are 1 and 4.

$y = a(x - 1)(x - 4)$

The $y$ - is 4.

$4 = a(0 - 1)(0 - 4)$

$4 = 4a$

$1 = a$

So the function is

$y = (x - 1)(x - 4)$

$y = x^2 - 5x + 4$

*b.*

The $x$-intercepts are 0 and 6.
$y = ax(x - 6)$
The vertex is (3, 9).
$9 = a(3)(3 - 6)$
$9 = -9a$
$-1 = a$
So the function is
$y = -x(x - 6)$
$y = -x^2 + 6x$

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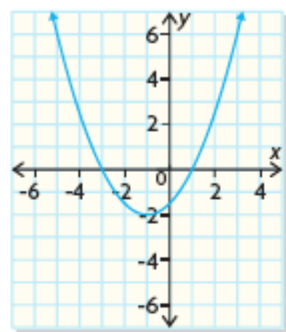
**Example**Identify the key characteristics you could use to determine the quadratic function that defines this graph. Explain how you would use these characteristics.




***Solution:***
e.g., The $x$-intercepts are –3 and 1.

The function is $y = a(x + 3)(x - 1)$.

Substitute a point on the graph such as (3, 6) to

determine that $a = \frac{1}{3}$

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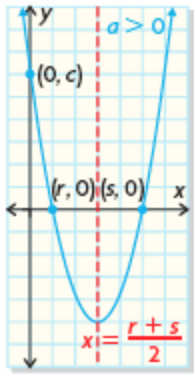
## Summary of Factored Form of Quadratic Form

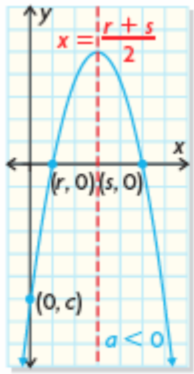
A quadratic function that is written in the form

$y = a(x - r)(x - s)$

has the following characteristics:

- The $x$-intercepts of the graph of the function are $x = r$ and $x = s$.
- The linear equation of the axis of symmetry is $x = \frac{r + s}{2}$.
- The y-intercept, c, is $c = a \cdot r \cdot s$.




• If a quadratic function has only one *x*-intercept, the factored form can be written as follows:

$f(x) = a(x - r)(x - r)$

$f(x) = a(x - r)^2$